ATM 241, Spring 2020
Lecture 9b
The Wind-Driven Circulation (Part 2)

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Marshall & Plumb
Ch. 10

In this section…

Questions

- What is the relationship between wind stress and Ekman pumping / suction?
- What is responsible for strong western boundary currents?
- How does wind stress drive the equatorial counter-current?

Ekman Pumping and Suction

Ekman Pumping and Suction

Recall that the continuity equation in the ocean is given by

$$\nabla \cdot \mathbf{u} = 0$$

Expand: $$\nabla_h \cdot \mathbf{u}_h + \frac{\partial w}{\partial z} = 0$$

Use zero horizontal divergence of geostrophic motion: $$\nabla_h \cdot \mathbf{u}_{ag} + \frac{\partial w}{\partial z} = 0$$

And that the ageostrophic winds are associated with friction in the Ekman layer:

$$f\mathbf{k} \times \mathbf{u}_{ag} = \frac{1}{\rho_{\mathrm{ref}}} \frac{\partial \tau}{\partial z} \quad \Rightarrow \quad \mathbf{u}_{ag} = -\frac{1}{f\rho_{\mathrm{ref}}} \mathbf{k} \times \frac{\partial \tau}{\partial z}$$

And so: $$w_{ek} = \frac{1}{\rho_{\mathrm{ref}}} \mathbf{k} \cdot \nabla \times \left(\frac{\tau_{wind}}{f} \right)$$

Ekman Pumping and Suction

$$w_{ek} = \frac{1}{\rho_{\mathrm{ref}}} \mathbf{k} \cdot \nabla \times \left(\frac{\tau_{wind}}{f} \right)$$

Rotation in the wind stress then drives vertical motion. In particular, an (anticyclonic) cyclonic applied stress leads to (downwelling) upwelling.

Anticyclonic stress:
Ekman pumping (negative w_{ek})

Cyclonic stress:
Ekman suction (positive w_{ek})

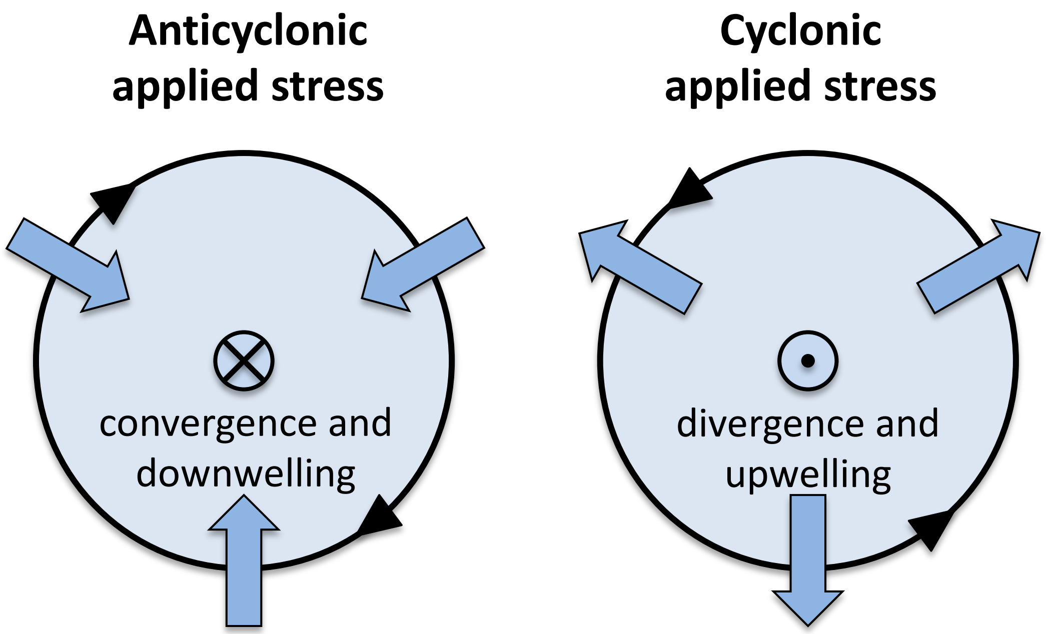


Figure: Similar to the atmosphere, vortical wind stresses are responsible for driving horizontally divergent motion, which in turn drive upwelling and downwelling.

Ekman Pumping and Suction

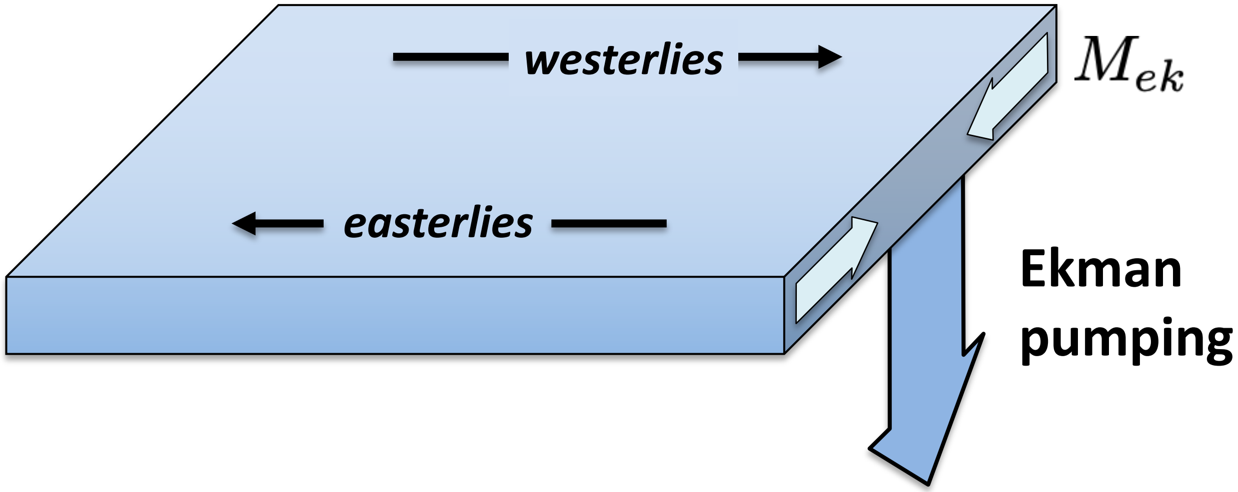
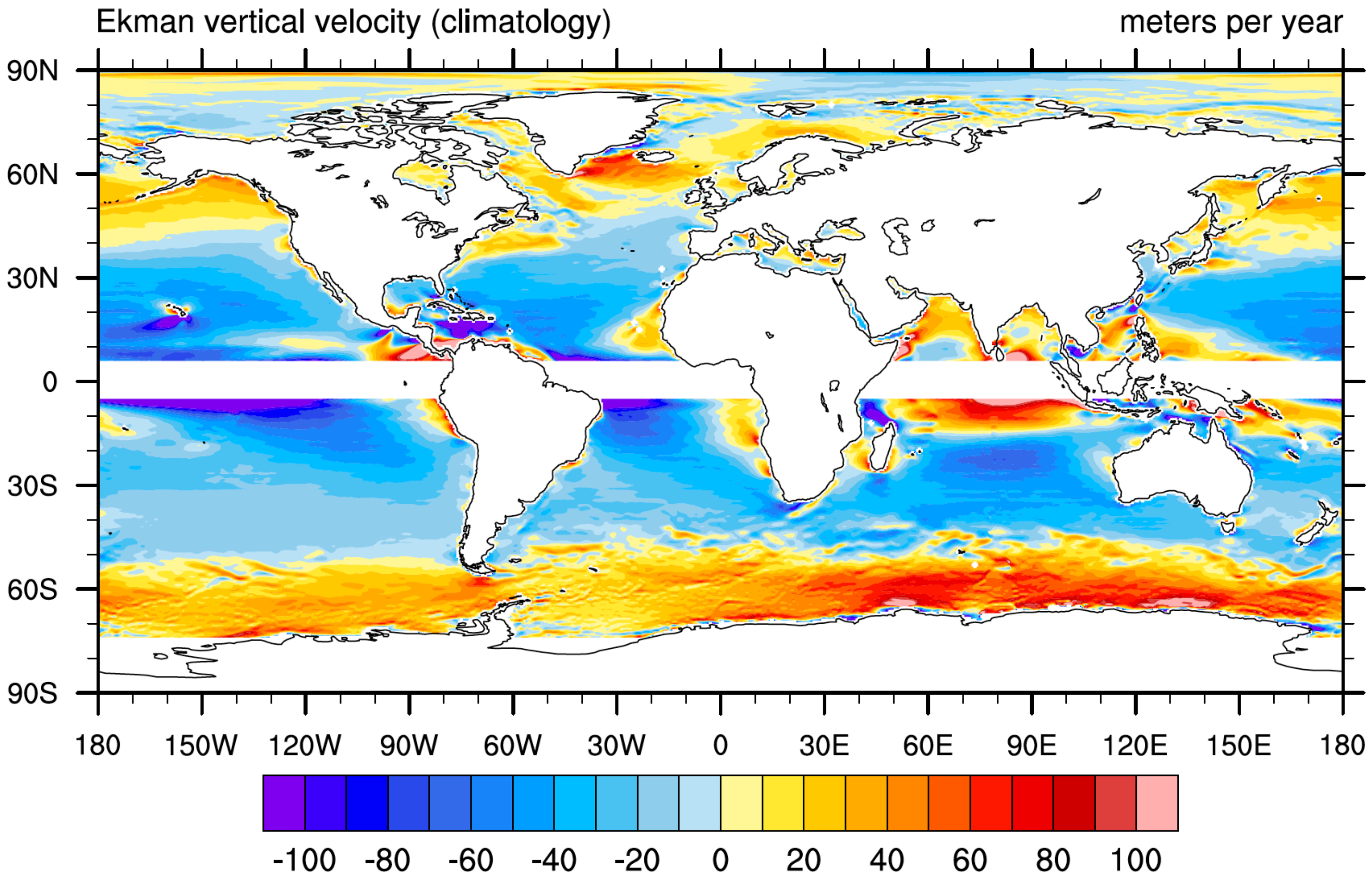


Figure: A schematic showing midlatitude westerlies (eastward wind stress) and tropical easterlies (westward wind stress). The resulting circulation is anti-cyclonic in the northern hemisphere and hence responsible for downward pumping of surface waters into the interior ocean.

Ekman Pumping and Suction

Figure: The global pattern of Ekman vertical velocity (m/y) using the mean observed wind stress. Observe regions of upwelling / downwelling.

This figure does not include the equatorial band, since f ≈ 0 here and the Ekman theory does not apply.



Equatorial Upwelling

Consider the governing equations of the Ekman layer in the vicinity of the equator:

$$-fv + \frac{1}{\rho_{\text{ref}}}\frac{\partial p}{\partial x} = \frac{1}{\rho_{\text{ref}}}\frac{\partial \tau_x}{\partial z}$$
$$fu + \frac{1}{\rho_{\text{ref}}}\frac{\partial p}{\partial y} = \frac{1}{\rho_{\text{ref}}}\frac{\partial \tau_y}{\partial z}$$

Recall the Coriolis force is given by $f = 2\Omega \sin\phi$

We have previously approximated the Coriolis force as constant, but this approximation cannot be used to explain equatorial upwelling. Instead, we write

$$f = 2\Omega \sin\phi \approx \beta y$$

where $$\beta = \frac{2\Omega}{a}\cos\phi = 2.28 \times 10^{-11}\ \text{m}^{-1}\ \text{s}^{-1}$$

Beta plane

Equatorial Upwelling

The zonal momentum equation then reads

$$-\beta y v = \frac{1}{\rho_{\text{ref}}}\left(-\frac{\partial p}{\partial x} + \frac{\partial \tau_x}{\partial z}\right)$$

If we assume zonal symmetry $\frac{\partial p}{\partial x} = 0$

and so over the depth of the Ekman layer

$$\langle v \rangle \approx -\frac{\tau_{wind_x}}{\beta y \rho_{\text{ref}}}$$

Near the equator $\tau_{wind_x} < 0$ and so

$$\begin{cases} \langle v \rangle < 0 & \text{if } y < 0, \\ \langle v \rangle > 0 & \text{if } y > 0. \end{cases}$$

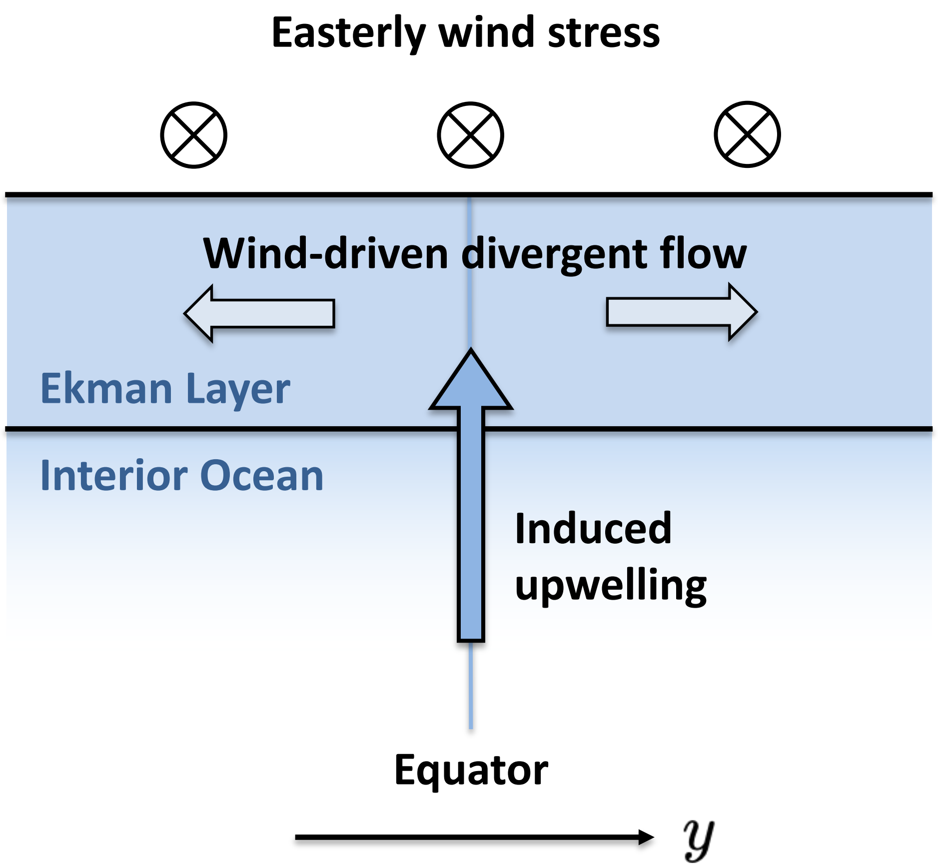


Figure: Schematic meridional cross section of the near-equatorial upwelling induced by westward wind stress at the equator.

Equatorial Upwelling

The global pattern of Ekman vertical velocity appears to the right using the mean observed wind stress and Ekman theory.

The equatorial strip is a region of upwelling since the trade winds drive fluid away from the equator in the surface layer. This process then requires a supply of water from below.

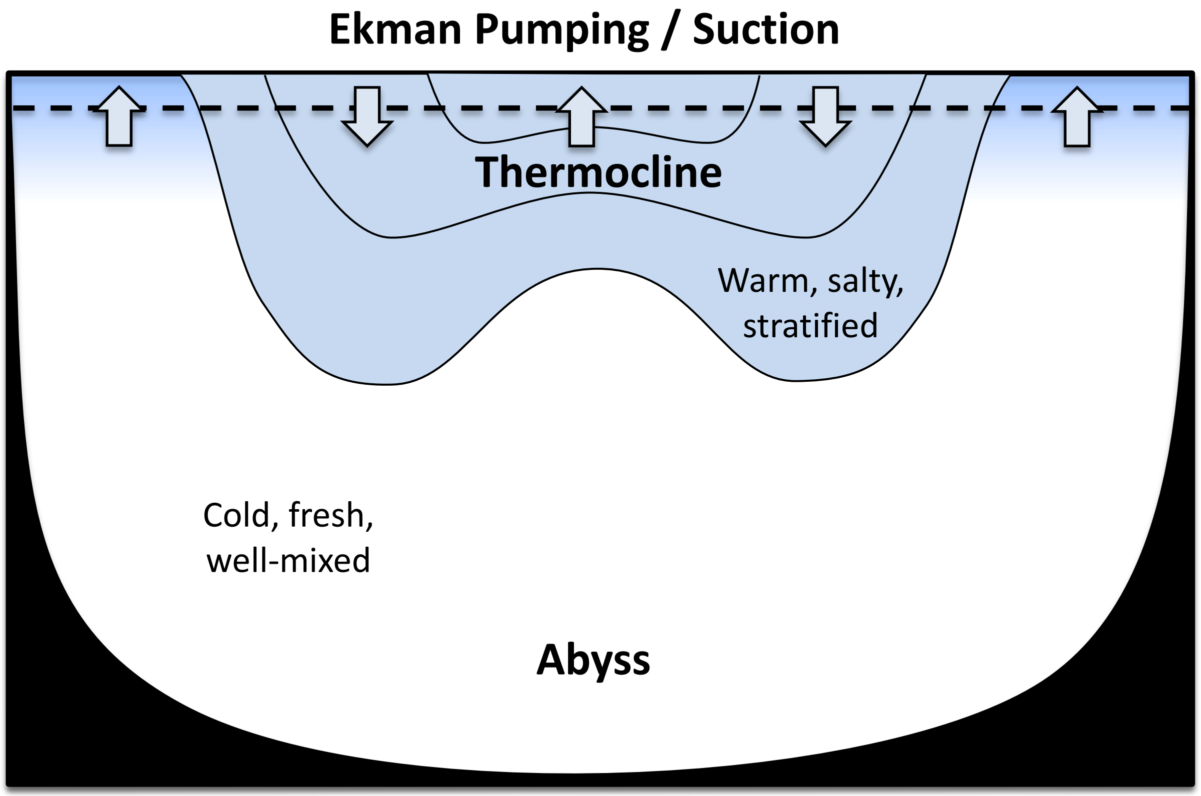
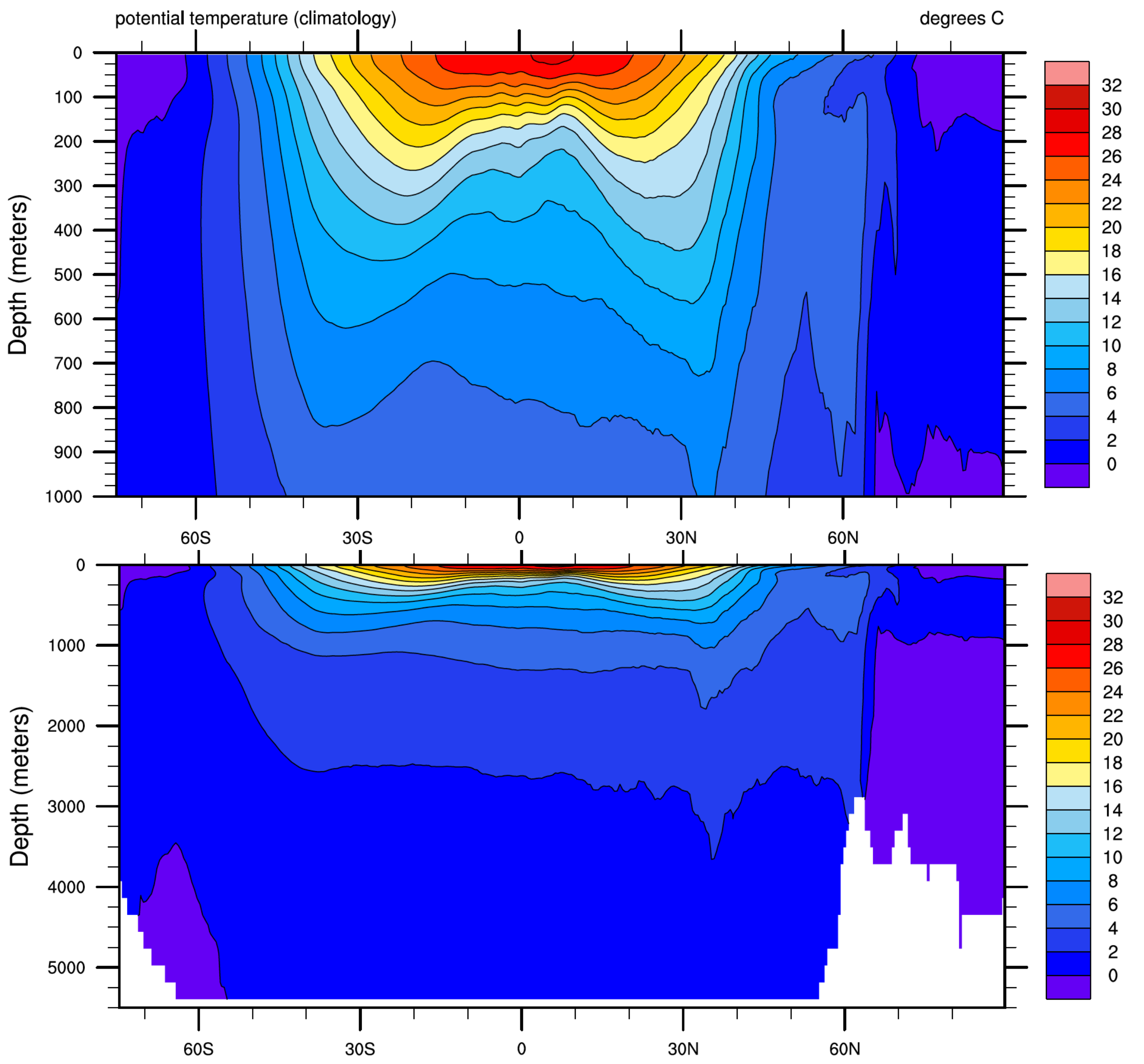


Figure: The direction of Ekman pumping and suction is responsible for the odd bi-modal shape of the ocean's density anomaly.

Temperature

Figure: Zonal average annual-mean potential temperature in the world oceans.

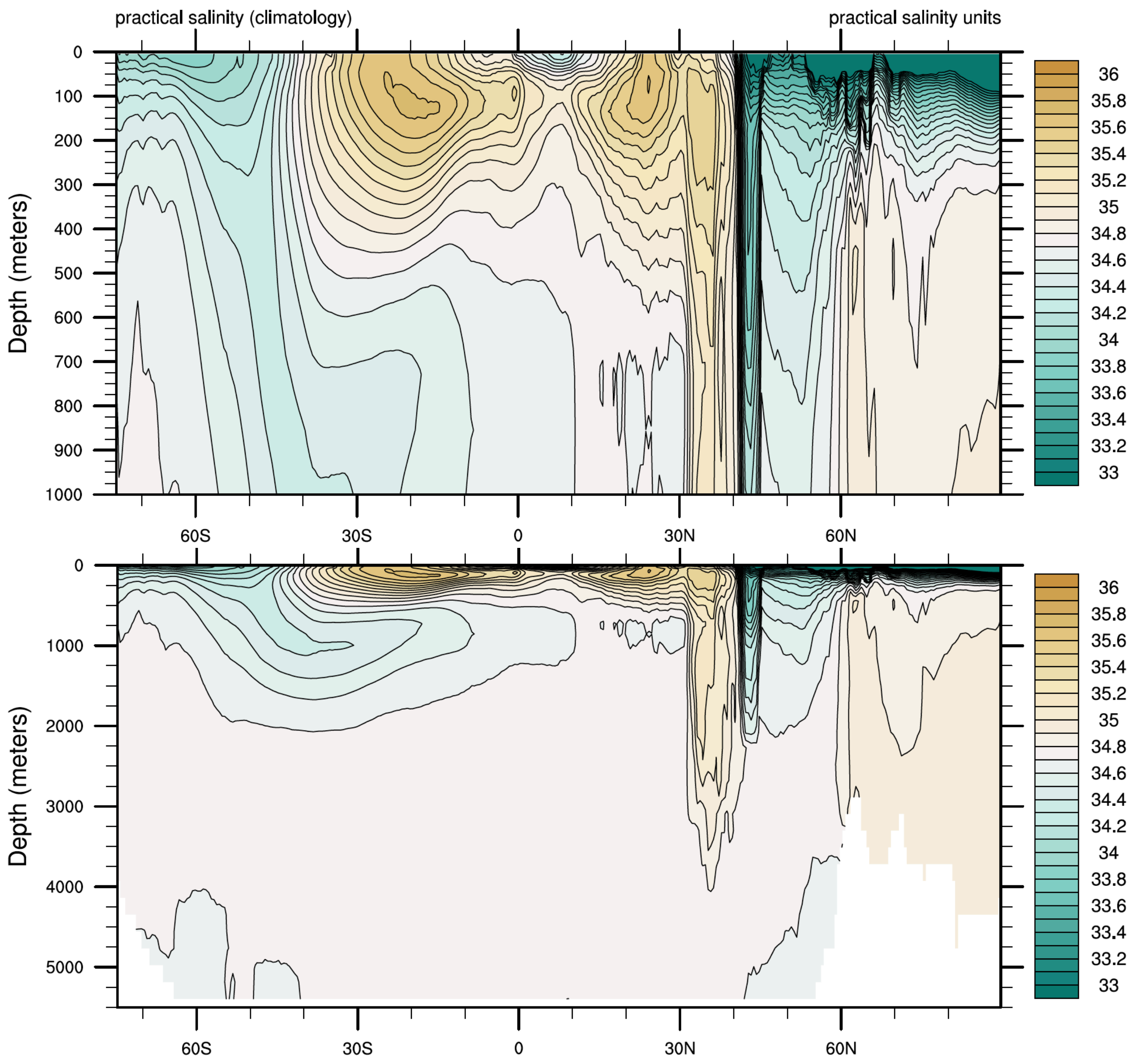
Temperatures follow atmospheric temperatures at the near-surface, but are largely uniform at depths below 1000 meters.



Salinity

Figure: Zonal average annual-mean salinity in the world oceans.

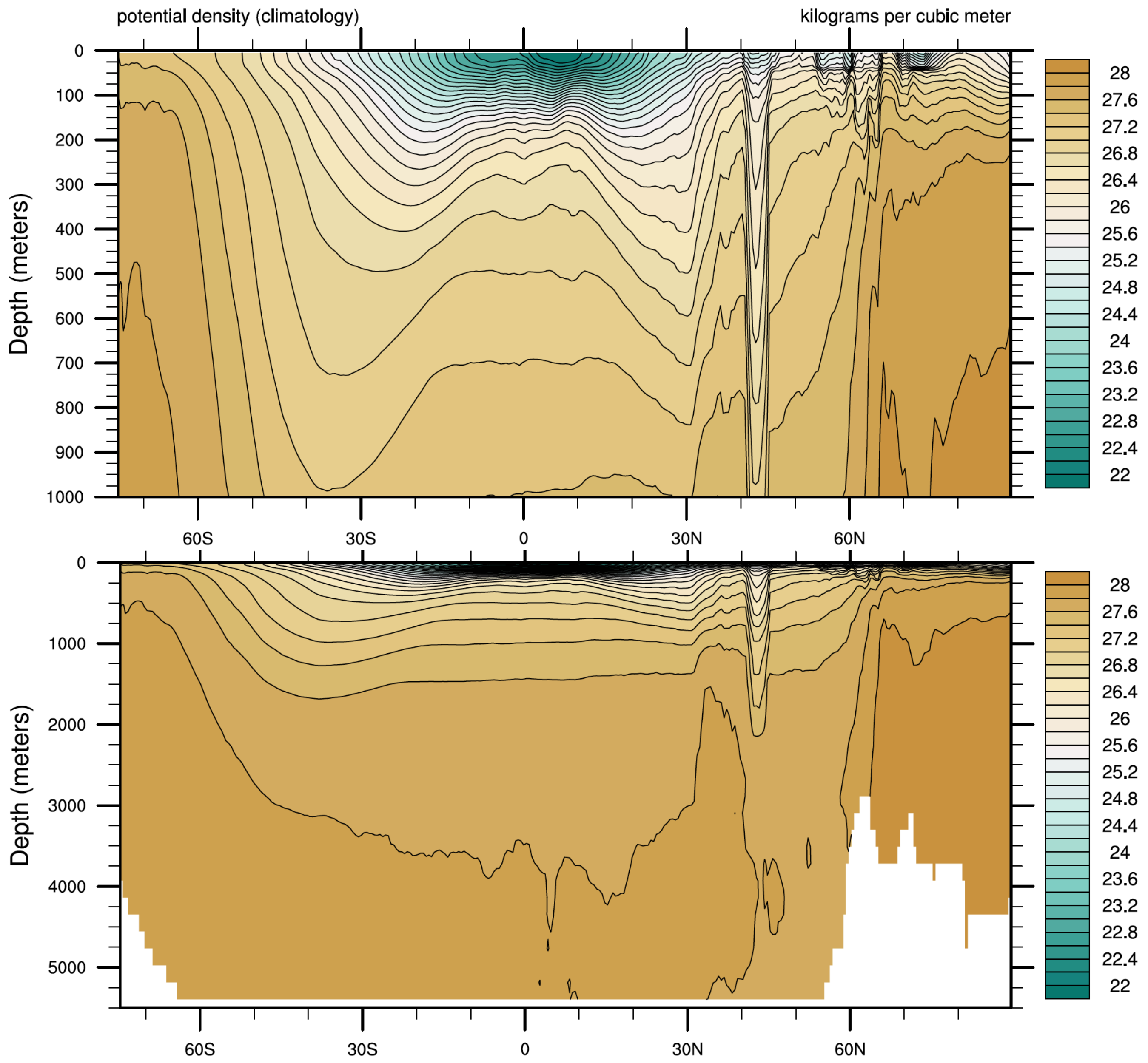
The spike in salinity with depth driven by averaging over the Mediterranean.



Potential Density

Figure: Zonal average annual-mean potential density anomaly in the world oceans. Note that darker colors indicate less dense fluid. Compare with zonal average annual-mean temperature.

$$\sigma = \rho - \rho_{ref}$$



Interior Ocean Response

We are now interested in understanding how the Ekman layer impacts the geostrophic flow in the interior ocean. Since Ekman pumping / suction impacts the horizontal divergence of the flow, it is natural to examine this quantity:

$$\nabla_h \cdot \mathbf{u}_g = \frac{\partial}{\partial x}\left(-\frac{1}{\rho_{\text{ref}} f}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho_{\text{ref}} f}\frac{\partial p}{\partial x}\right) = -\frac{\beta}{f} v_g$$

Since f varies with latitude, the horizontal geostrophic flow has a non-zero divergence on large scales.

$$f = 2\Omega \sin\phi \approx \beta y$$

where $$\beta = \frac{2\Omega}{a}\cos\phi = 2.28 \times 10^{-11}\ \mathrm{m}^{-1}\ \mathrm{s}^{-1}$$

Beta plane

Interior Ocean Response

The horizontal divergence is then associated with stretching of fluid columns caused by Ekman pumping / suction via the continuity equation:

$$\nabla_h \cdot \mathbf{u}_g = -\frac{\partial w}{\partial z}$$

Then combining with $\nabla_h \cdot \mathbf{u}_g = -\frac{\beta}{f} v_g$

$$\beta v_g = f \frac{\partial w}{\partial z}$$

So on the large-scale, stretching in vertical columns **must induce an associated meridional velocity**.

Interior Ocean Response

If vertical velocities in the abyss are much smaller than vertical velocities near the surface caused by Ekman pumping, we can integrate

$$\beta v_g = f\frac{\partial w}{\partial z}$$

And so obtain

$$\beta\langle v_g\rangle = f\frac{w_{Ek}}{h}$$

Hence Ekman pumping (in the midlatitudinal gyres: $w_{Ek} < 0$) naturally drives **an equatorward circulation** as a consequence of the variation in f.

Interior Ocean Response

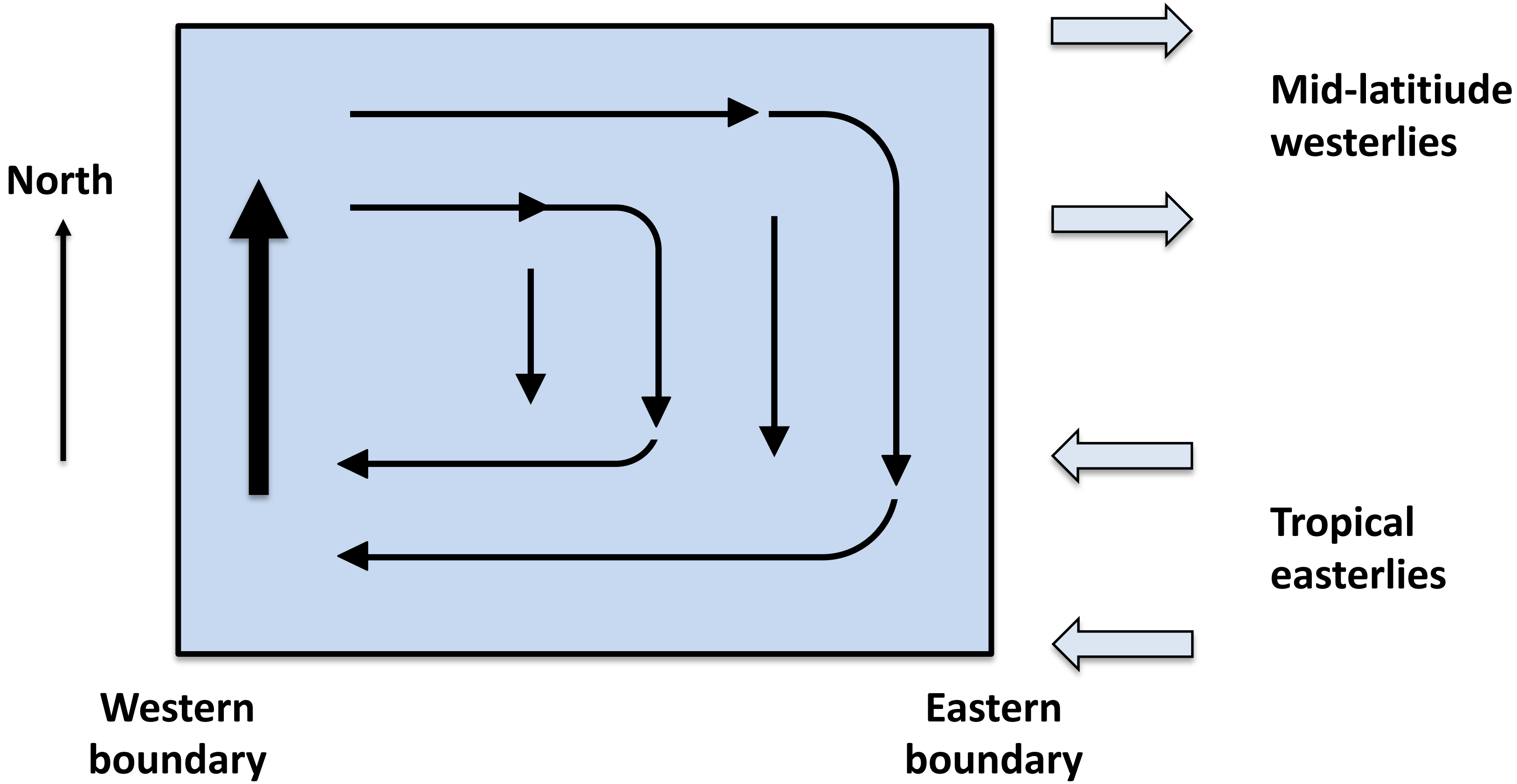


Figure: Since the large-scale impact of Ekman pumping drives equatorward motion, the Western boundary currents must be enhanced to satisfy mass balance. Correspondingly, Eastern boundary currents are weakened.

Interior Ocean Response

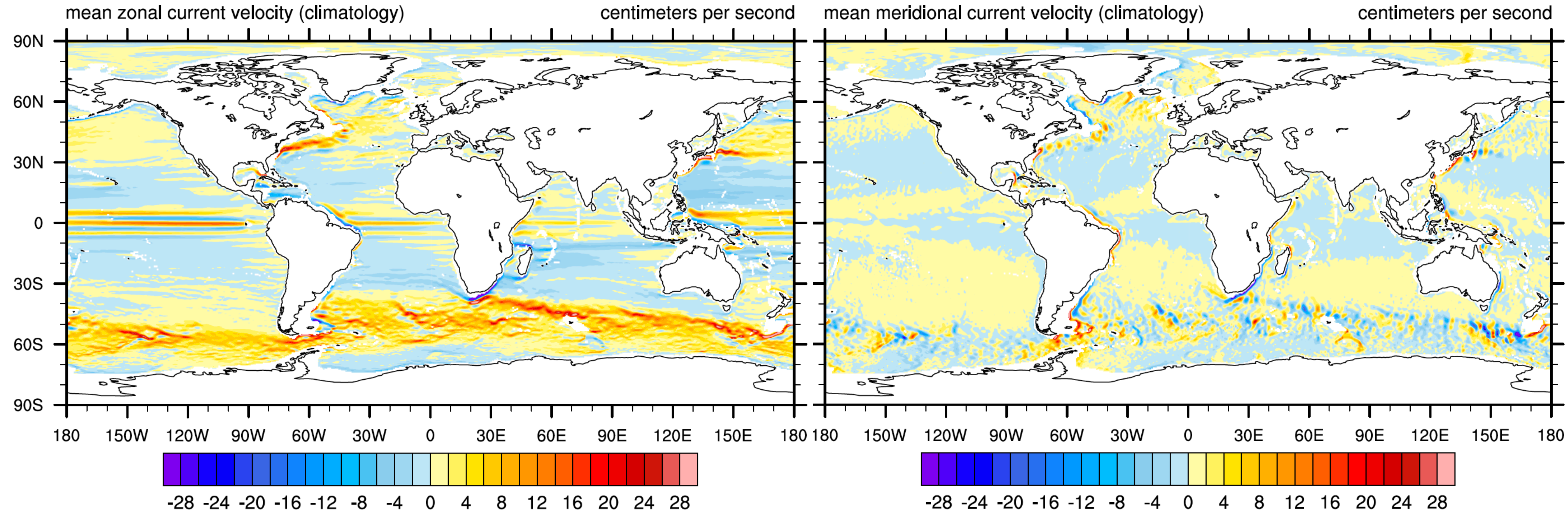


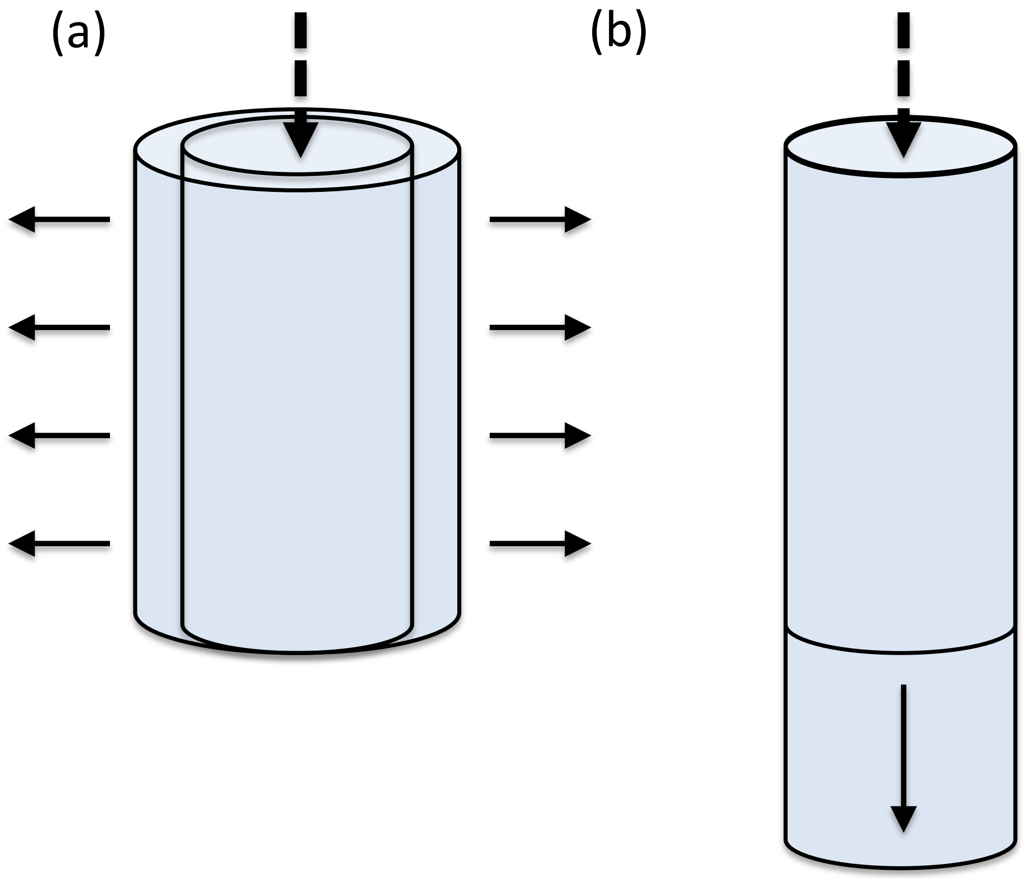
Figure: Time mean (left) zonal velocity and (right) meridional velocity integrated over the top 1000 meters of the ocean.

Taylor Column Interpretation

Recall that for incompressible flow, velocity cannot vary along the direction of the rotation axis. This leads to the existence of **Taylor columns**.

However, Ekman pumping leads to convergence at the top of the fluid column which in turn pumps additional fluid into the column. This must lead to a **change in volume** of the fluid column.

Figure: Two mechanisms by which the volume of the fluid column can change.

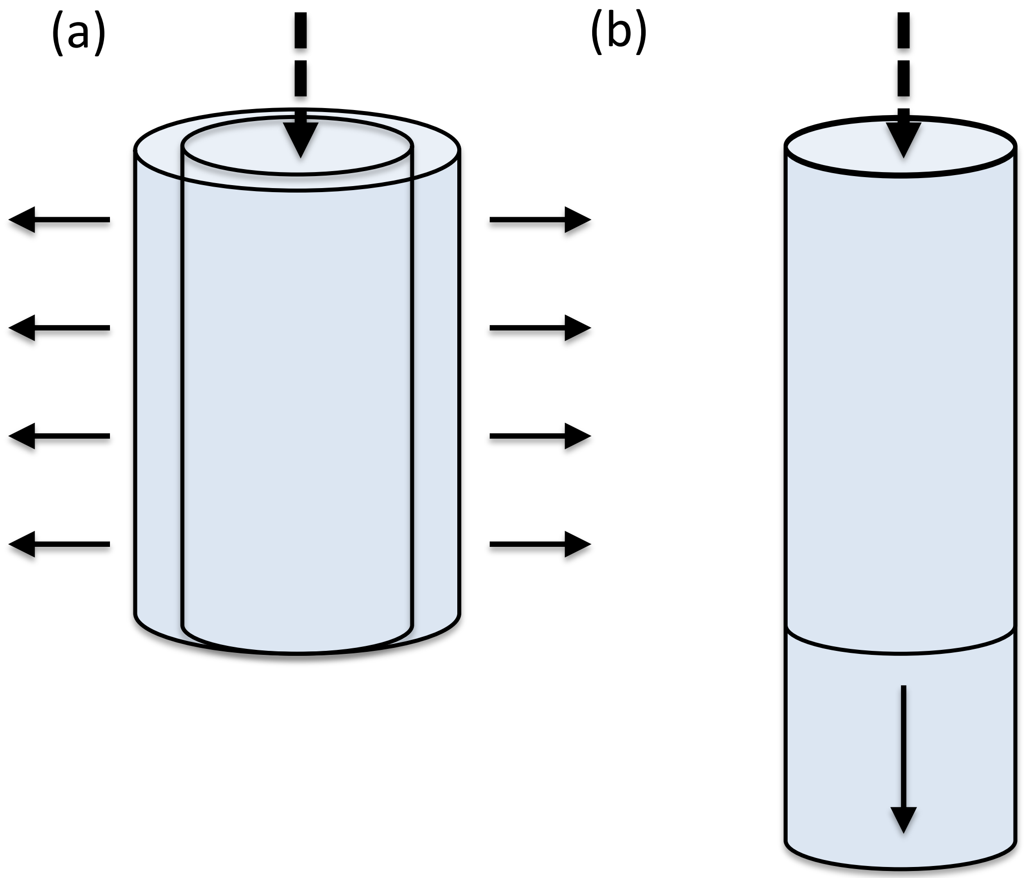


Taylor Column Interpretation

Figure: Two mechanisms by which the volume of the fluid column can change.

(a) However, if Ekman pumping caused an increase in the radius of the fluid column then conservation of angular momentum would eventually require the angular velocity to asymptote to zero.

(b) So it must be the case that Ekman pumping instead drives an increase in the height of the fluid column. How can this occur?

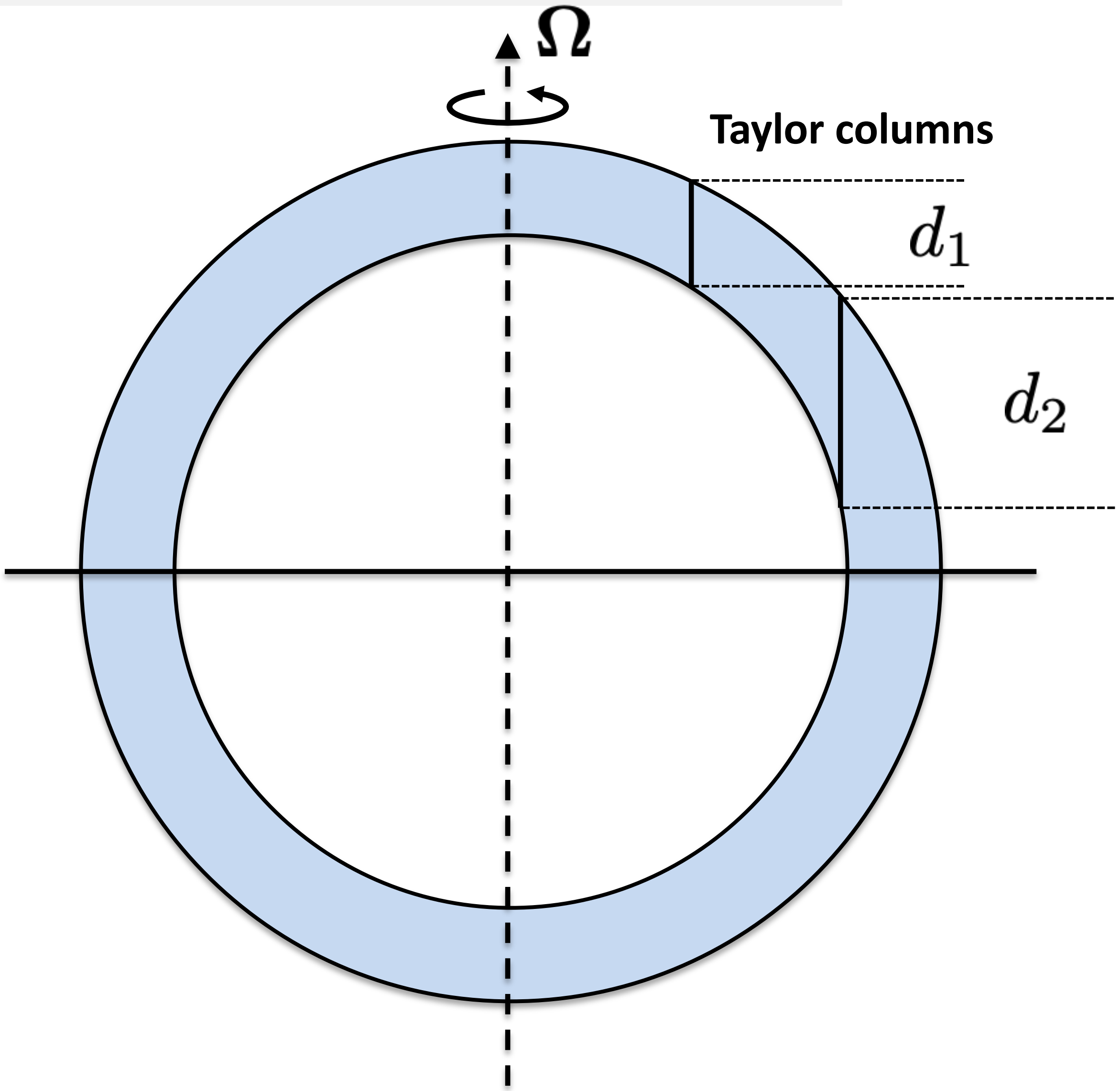


Taylor Column Interpretation

Figure: Since the top and bottom boundary are fixed, the only way that the Taylor column can be "stretched" in the vertical is if it moves from a higher latitude (d_1) to a lower latitude (d_2).

Since the Earth is spherical, columns aligned with the rotation axis are inherently longer as one nears the equator.

This provides an alternative explanation as to why Ekman pumping drives an equatorward circulation.



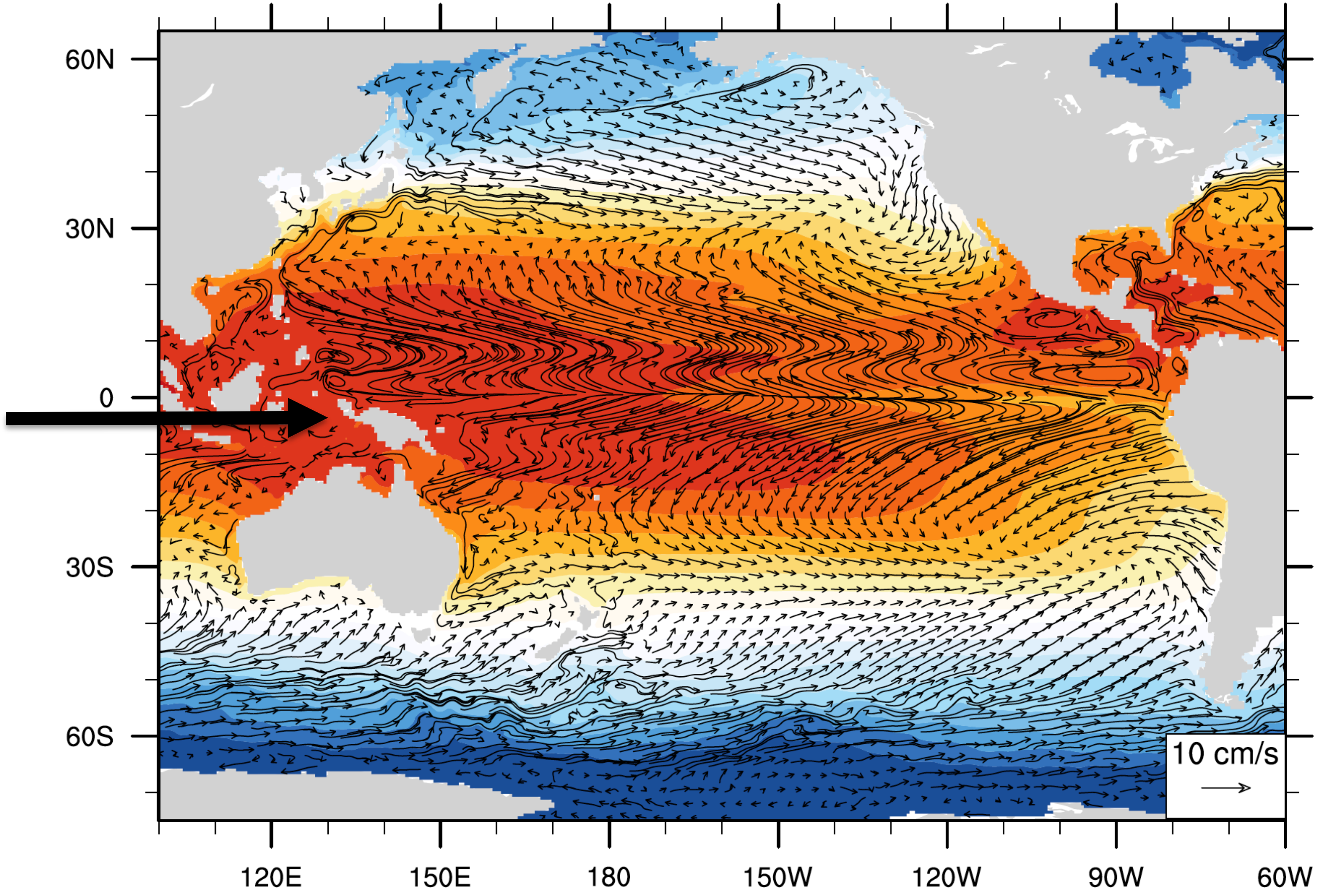
Sverdrup Theory

Pacific Ocean Surface Currents

Figure: Major surface currents in the Pacific Ocean from SODA3.

Observe equatorial current and midlatitudinal currents driven by prevailing winds. Also observe equatorial counter-current (against prevailing winds).

The counter-current driven by zonal temperature gradients in the tropics (see El Niño-Southern Oscillation, ENSO).



Equatorial Ekman Pumping and Suction

The goal of this section is to explain the existence of the Equatorial Counter-Current (ECC) using our knowledge of the ocean system.

Recall that for constant f, the geostrophic wind is divergence free:

$$\nabla \cdot \mathbf{u}_g = 0$$

For constant f

However, f is not constant on the surface of the sphere:

$$f = 2\Omega \sin\phi \approx f_0 + \beta y$$

Taylor expansion about ϕ_0

with $$f_0 = 2\Omega \sin\phi_0 \qquad \beta = \frac{2\Omega\cos\phi_0}{a}$$

Equatorial Ekman Pumping and Suction

$$f = 2\Omega \sin\phi \approx f_0 + \beta y$$

$$\nabla_h \cdot \mathbf{u}_g = \frac{\partial}{\partial x}\left(-\frac{1}{\rho_{\mathrm{ref}} f}\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho_{\mathrm{ref}} f}\frac{\partial p}{\partial x}\right) = -\frac{\beta}{f} v_g$$

Continuity Equation: $$\nabla_h \cdot \mathbf{u}_g + \frac{\partial w}{\partial z} = 0$$

$$\beta v_g = f \frac{\partial w}{\partial z}$$

Sverdrup Theory

Return to the momentum equation:

$$-fv + \frac{1}{\rho_{\text{ref}}}\frac{\partial p}{\partial x} = \frac{1}{\rho_{\text{ref}}}\frac{\partial \tau_x}{\partial z}$$

$$fu + \frac{1}{\rho_{\text{ref}}}\frac{\partial p}{\partial y} = \frac{1}{\rho_{\text{ref}}}\frac{\partial \tau_y}{\partial z}$$

What is this?

Cross-differentiating and applying $\frac{\partial f}{\partial y} = \beta$ and $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z}$

$$\Rightarrow \quad \beta v = f\frac{\partial w}{\partial z} + \frac{1}{\rho_{\text{ref}}}\frac{\partial}{\partial z}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)$$

Sverdrup Theory

$$\beta v = f\frac{\partial w}{\partial z} + \frac{1}{\rho_{\text{ref}}}\frac{\partial}{\partial z}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)$$

Now take this equation and integrate from the bottom of the ocean (at z = -D) to the very top. At both the bottom and top of the ocean we have

$$w = 0$$

And so after some Calculus:

$$\beta V = \frac{1}{\rho_{\text{ref}}}\mathbf{k}\cdot\nabla\times\tau_{wind}$$

where $V = \int_{-D}^{0} v dz$

Sverdrup Relation

Sverdrup Theory

Sverdrup relationship: $$\beta V = \frac{1}{\rho_{\text{ref}}} \mathbf{k} \cdot \nabla \times \tau_{wind}$$

If the wind is purely zonal $\tau_{wind_y} = 0$ and so

$$\beta V = -\frac{1}{\rho_{\text{ref}}} \frac{\partial \tau_{wind_x}}{\partial y}$$

Where is meridional transport zero?

Sverdrup Theory

Continuity equation: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z}$$

$$\int_{-D}^{0} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] dz = -\int_{-D}^{0} \frac{\partial w}{\partial z} dz = -w(0) + w(-D) = 0$$

$$U = \int_{-D}^{0} u dz$$

$$V = \int_{-D}^{0} v dz$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

Sverdrup Theory

Sverdrup relationship: $$\beta V = \frac{1}{\rho_{\text{ref}}} \mathbf{k} \cdot \nabla \times \tau_{wind}$$

Since the flow is non-divergent we can write the bulk horizontal velocities in terms of a streamfunction:

$$U = -\frac{\partial \Psi}{\partial y} \quad V = \frac{\partial \Psi}{\partial x}$$

The streamfunction can then be obtained from the Sverdrup relationship:

$$\Psi(x,y) = \frac{1}{\rho_{\text{ref}}\beta} \int_{x_e}^{x} \mathbf{k} \cdot \nabla \times \tau_{\text{wind}} dx$$

Where x_e is the horizontal position of the eastern boundary.

Sverdrup Theory

Sverdrup relationship:

$$\beta V = \frac{1}{\rho_{\text{ref}}} \mathbf{k} \cdot \nabla \times \tau_{wind}$$

Sverdrup streamfunction:

$$\Psi(x,y) = \frac{1}{\rho_{\text{ref}}\beta} \int_{x_e}^{x} \mathbf{k} \cdot \nabla \times \tau_{\text{wind}} dx$$

where x_e is the horizontal position of the eastern boundary.

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$\tau_{wind_y} = 0$$

$$U = -\frac{1}{\rho_{\text{ref}}\beta} \int_{x_e}^{x} \frac{\partial^2 \tau_{wind_x}}{\partial y^2} dx$$

Recall: Surface Wind Stress

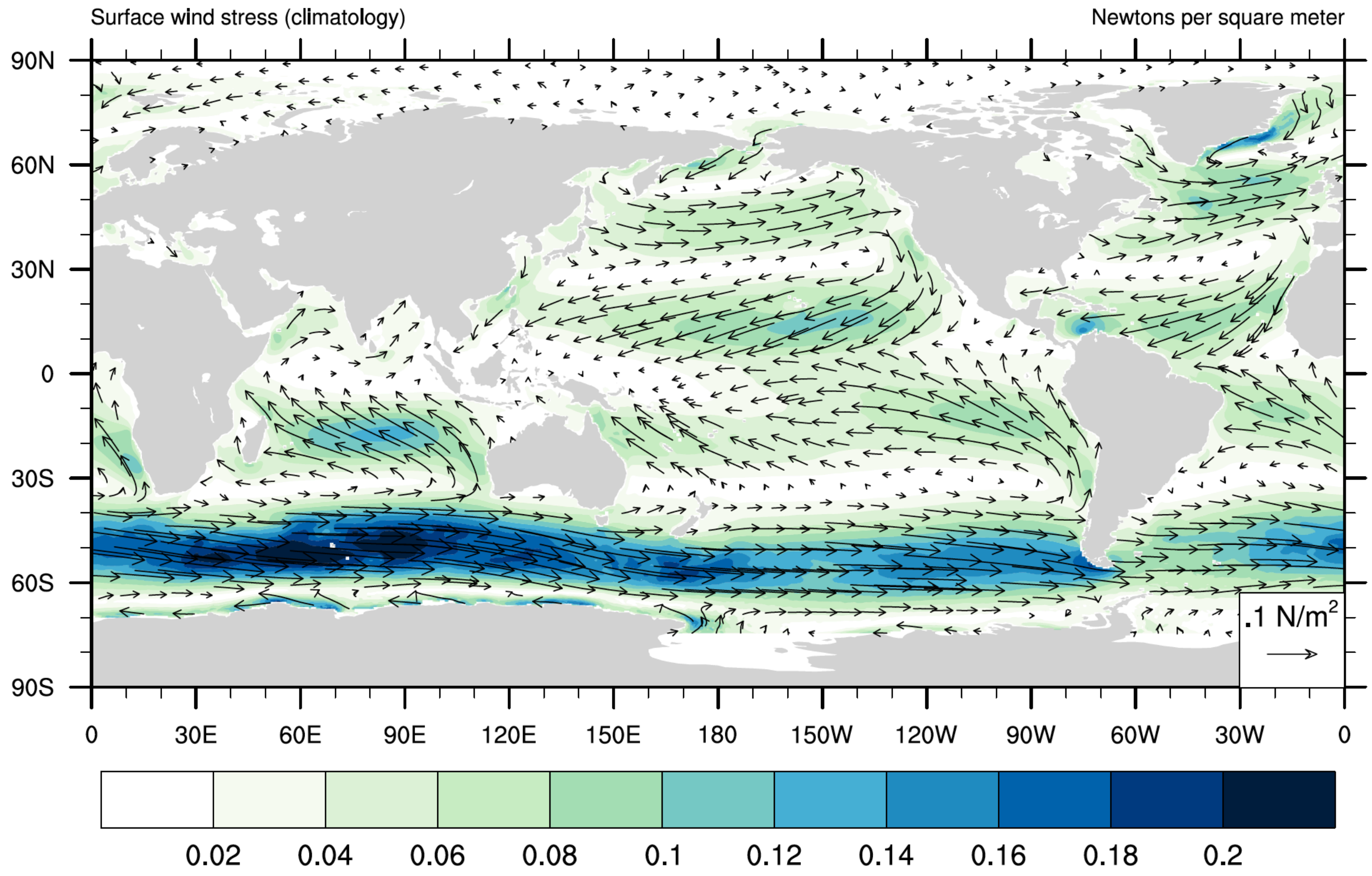


Figure: Surface wind stress from the atmosphere on the surface ocean (N/m^2).

Sverdrup Theory

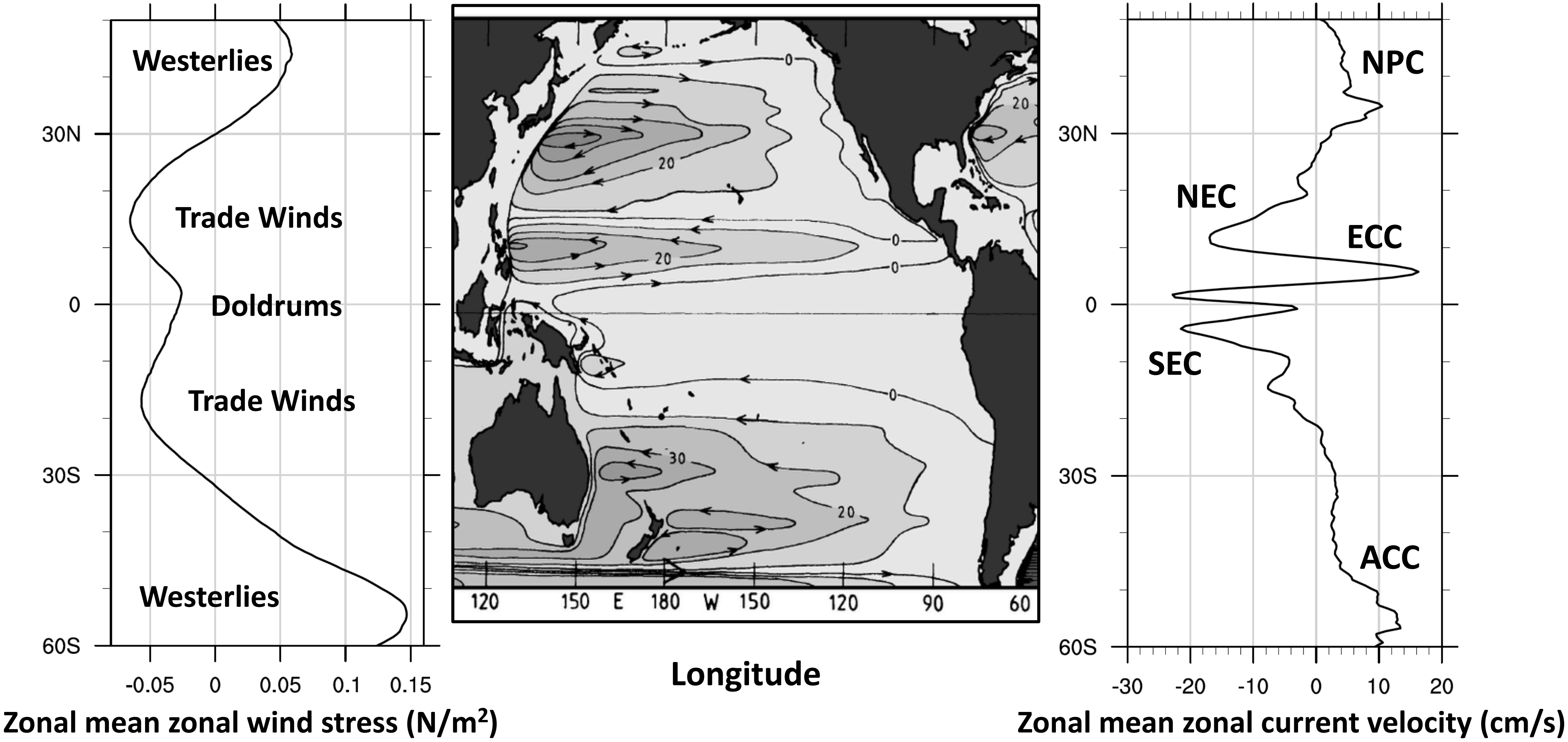


Figure: (Left) Zonally averaged zonal wind stress. **(Middle)** Sverdrup streamfunction from Tomczak and Godfrey "Regional Oceanography" Fig 4.7. **(Right)** Zonal current velocities obtained from surface drifters.

Sverdrup Theory

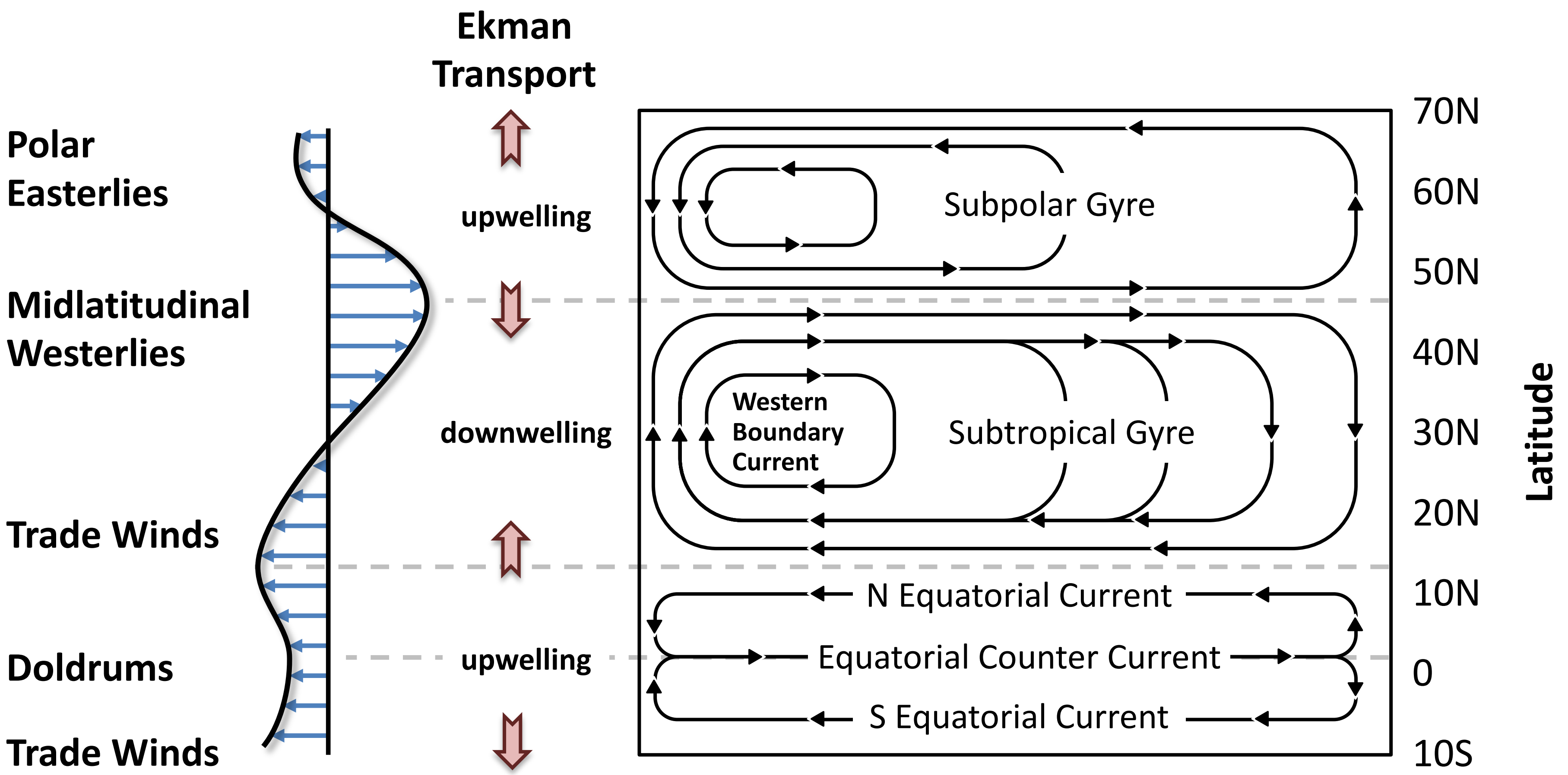


Figure: Schematic diagram showing the classification of ocean gyres and major ocean current systems and their relationship to prevailing winds.

ATM 241 Climate Dynamics
Lecture 9b
The Wind-Driven Circulation (Part 2)

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Thank You!

Equatorial Ekman Pumping and Suction

$$\beta v_g = f \frac{\partial w}{\partial z}$$

Depth-integrated mean

What does this equation mean?

- If vertical velocities in the abyss are much smaller than the surface Ekman pumping velocities, then northern hemisphere ocean currents will have a southward component in regions where $w_{Ek} < 0$ (Ekman pumping).

- Analogously, northern hemisphere ocean currents will have a northward component when $w_{Ek} > 0$ (Ekman suction).