ATM 241, Spring 2020
Lecture 9a
The Wind-Driven Circulation (Part 1)

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Marshall & Plumb
Ch. 10

In this section…

Definitions

- Wind stress
- Ocean eddy
- Wind-driven circulation
- Thermohaline circulation
- Surface wind stress

Questions

- What are the major drivers of ocean temperatures?
- What are the major drivers of ocean salinity?
- How important are ocean eddies to the circulation of the ocean?
- How does surface wind stress drive oceanic currents?
- Why is mass transport in the ocean at a 90 degree angle to the wind stress?

Ocean Temperature and Salinity

Sea Surface Temperature (^oC)

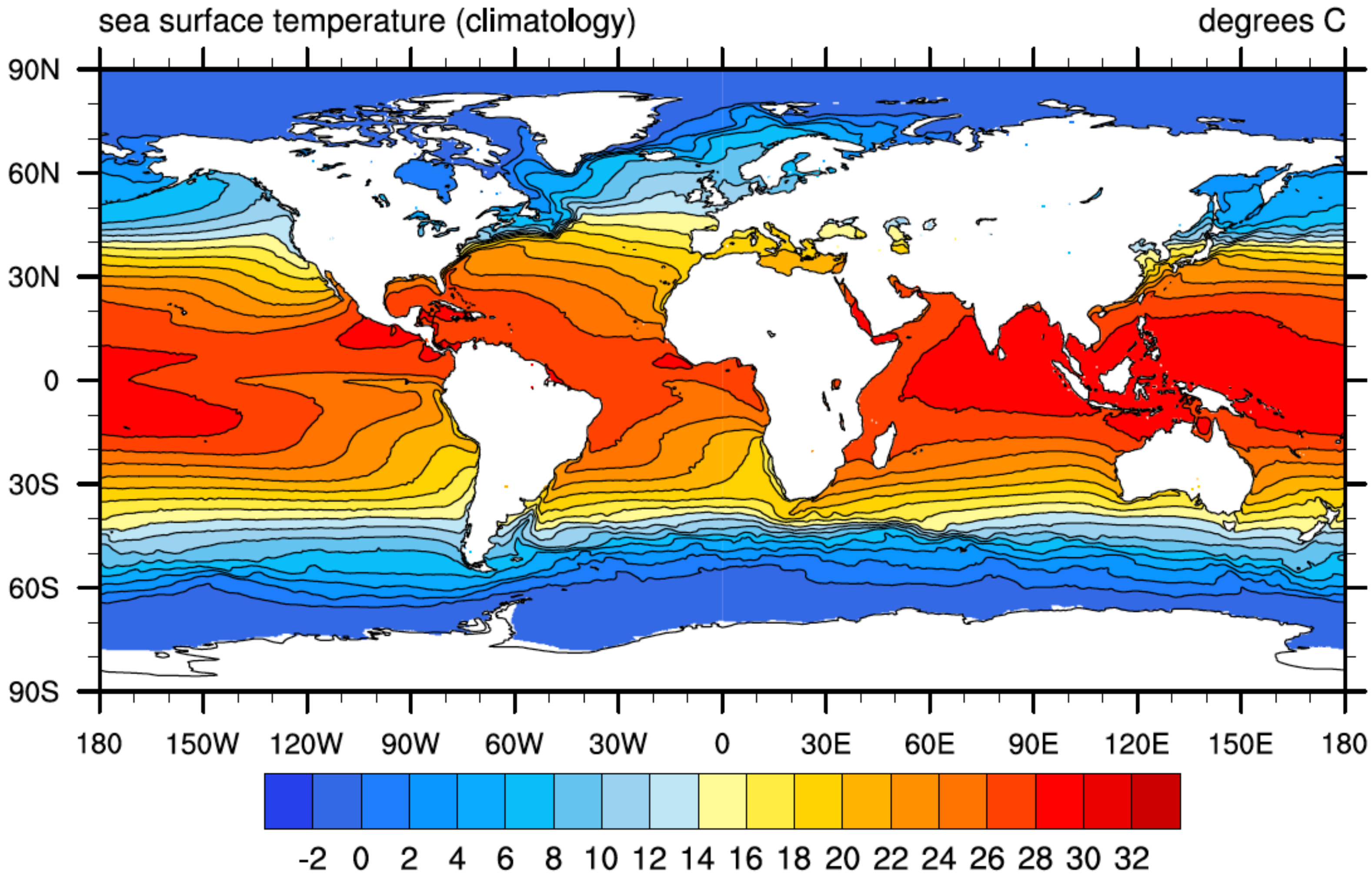


Figure: Average annual-mean sea-surface temperatures from ECMWF ERA5.

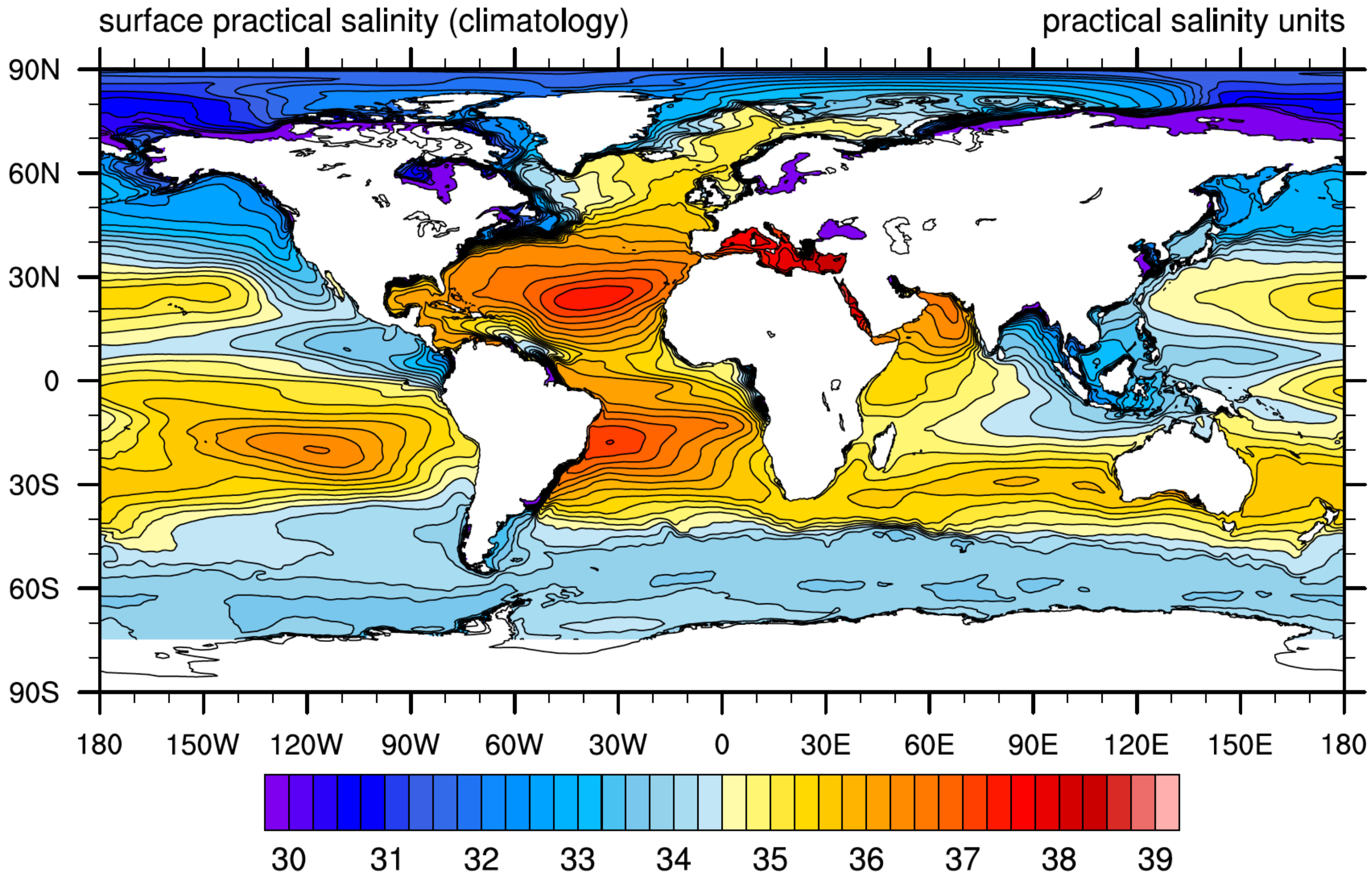
Drivers of Ocean Temperatures

Ocean surface temperatures are largely driven by **five factors**:

- **Surface warming:** In accordance with solar insolation.
- **Surface cooling:** Outgoing long-wavelength fluxes and sensible heat loss.
- **Evaporative cooling:** Also termed latent heat flux.
- **Meridional currents:** Strong poleward transport along the western edge of ocean basins, and equatorward transport throughout the interior.
- **Ocean upwelling:** Vertical exchange of ocean waters, especially in coastal regions.

Surface Salinity

Figure: Annual mean salinity distribution at the surface of the ocean (in PSU) from SODA3.



Drivers of Ocean Salinity

Ocean surface salinity is largely driven by **four factors**:

- **Evaporation:** Leads to an increase in local salinity as fresh water is removed from the ocean surface. Evaporation is closely related to the amount of solar radiation reaching the ocean surface.
- **Precipitation:** Leads to a decrease in local salinity as fresh water is added to the ocean surface.
- **River run-off:** Important for changing salinity in coastal regions as fresh water is mixed with ocean waters.
- **Brine rejection:** Results in increased salinity in polar regions as the formation of sea ice pulls fresh water from the ocean, resulting in increased salinity.

Drivers of Ocean Salinity

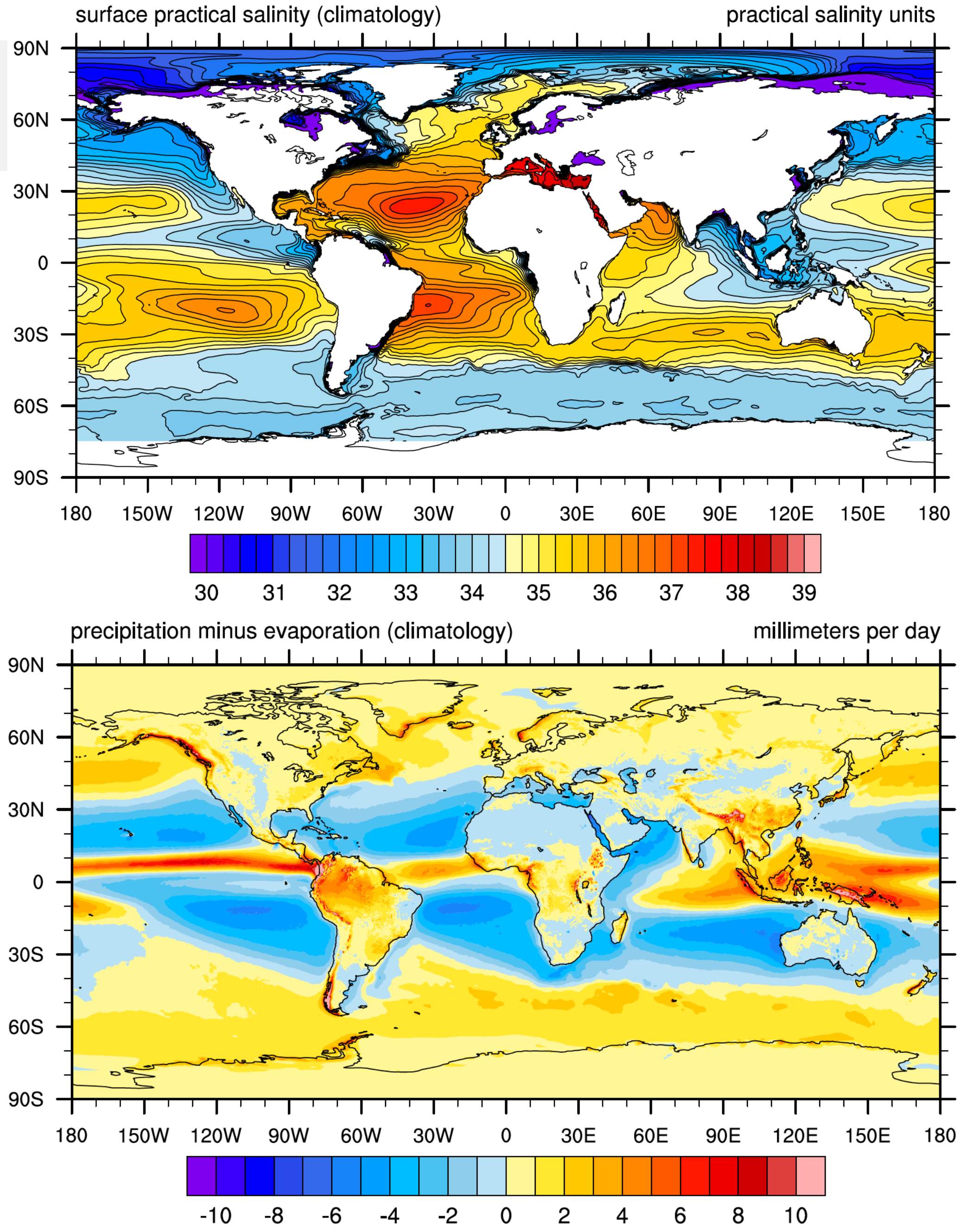


Figure: (top) Annual mean salinity distribution at the surface of the ocean (in PSU). (bottom) Annual mean precipitation minus evaporation (P-E, in mm/day).

There is a clear, strong correlation between P-E and ocean salinity. Regions of high evaporation tend to be salty, whereas regions of high precipitation (equatorial band, western Pacific) tend to be fresher.

Ocean Eddies

Ocean Currents

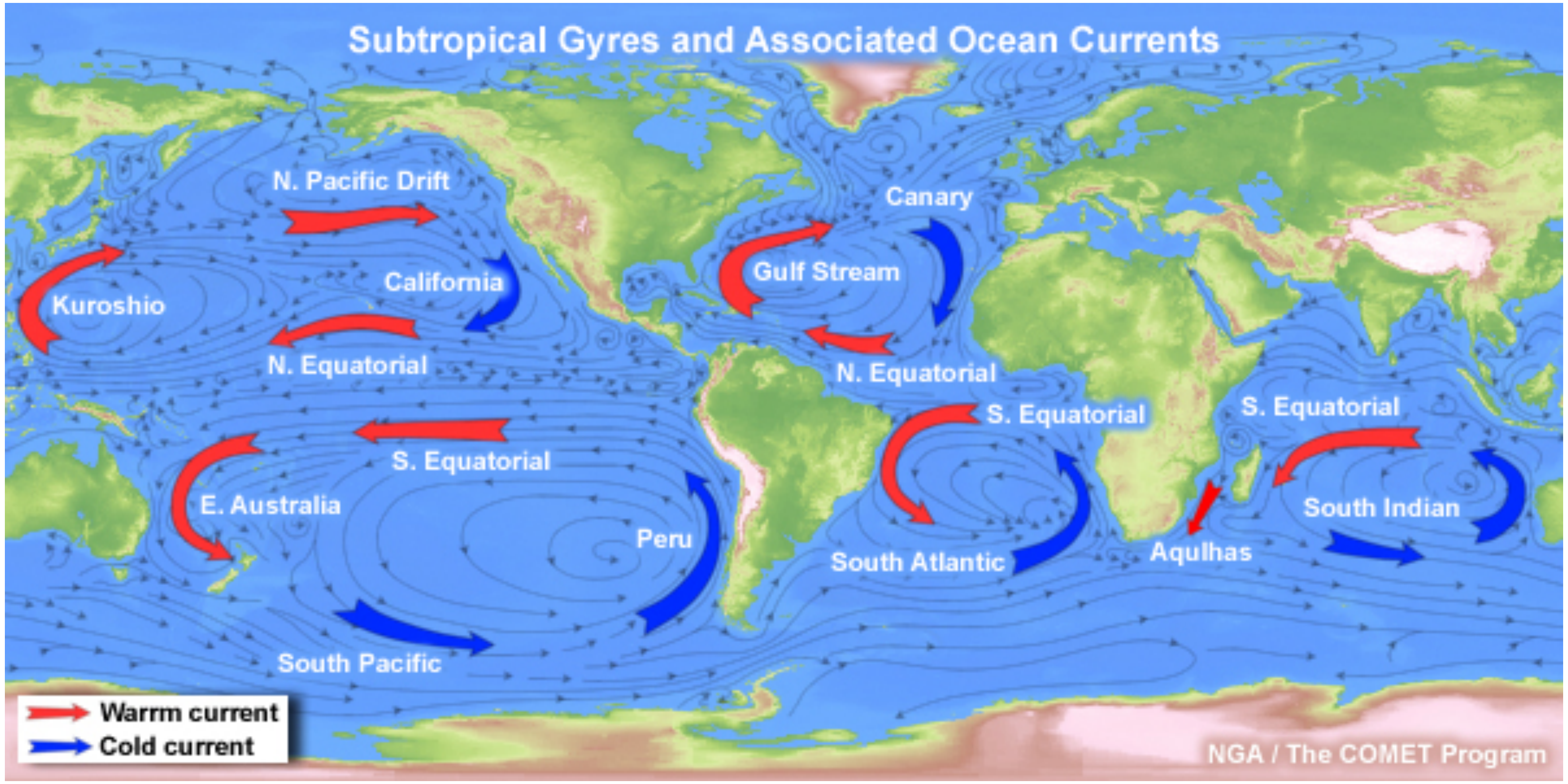


Figure: Subtropical gyres and associated ocean currents. (Source: NOAA) These currents represent average observations over 20 years of observations.

Ocean Eddies

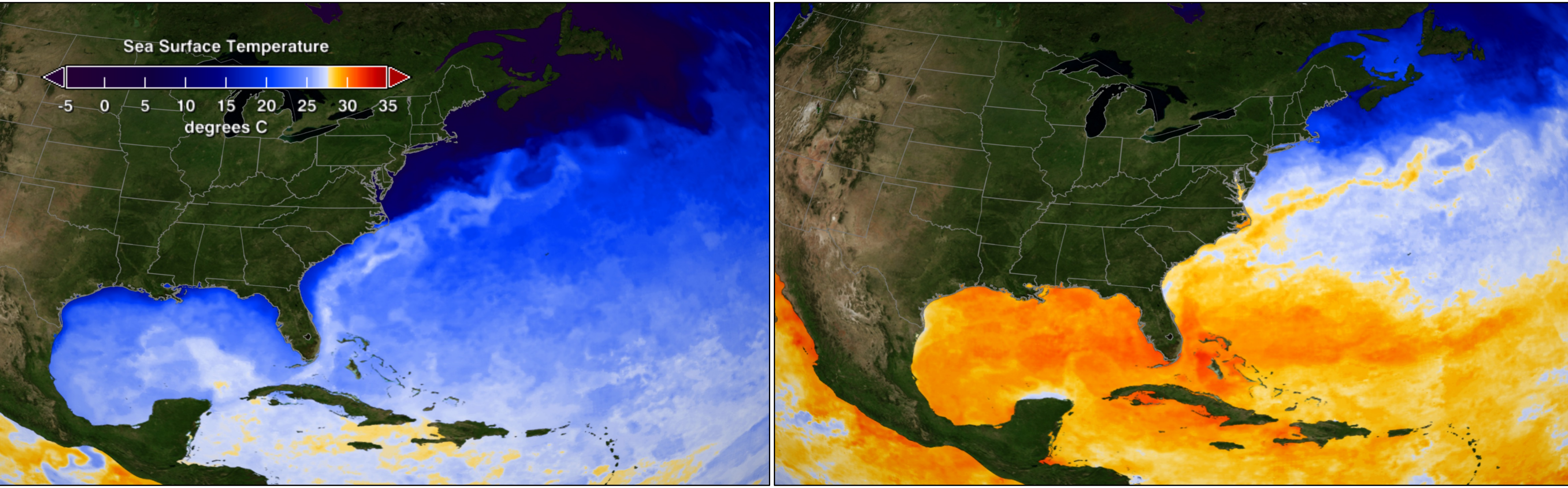
The description of oceanic circulation so far is generally inadequate for describing instantaneous circulation (much like knowledge of the general circulation of the atmosphere is inadequate for describing weather).

That is, the ocean is much more dynamic than the general circulation would suggest. Ocean eddies are prominent features of the ocean induced by instabilities from horizontal shear in the flow.

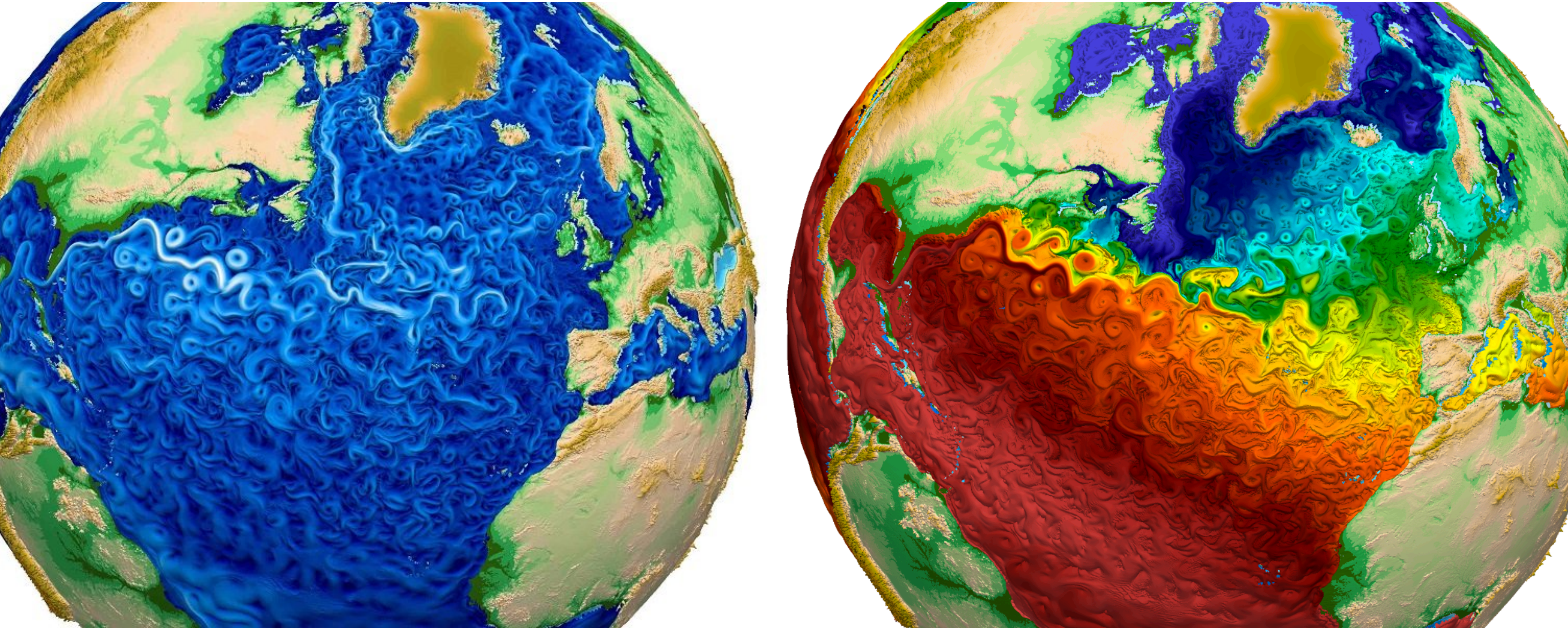
Definition: An **ocean eddy** is a circular current of water. These eddies emerge naturally from ocean currents that are “pinched off” and produce relatively local circulations of radius ~100km.

Sea Surface Temperatures

January 1st, 2008 August 1st, 2008

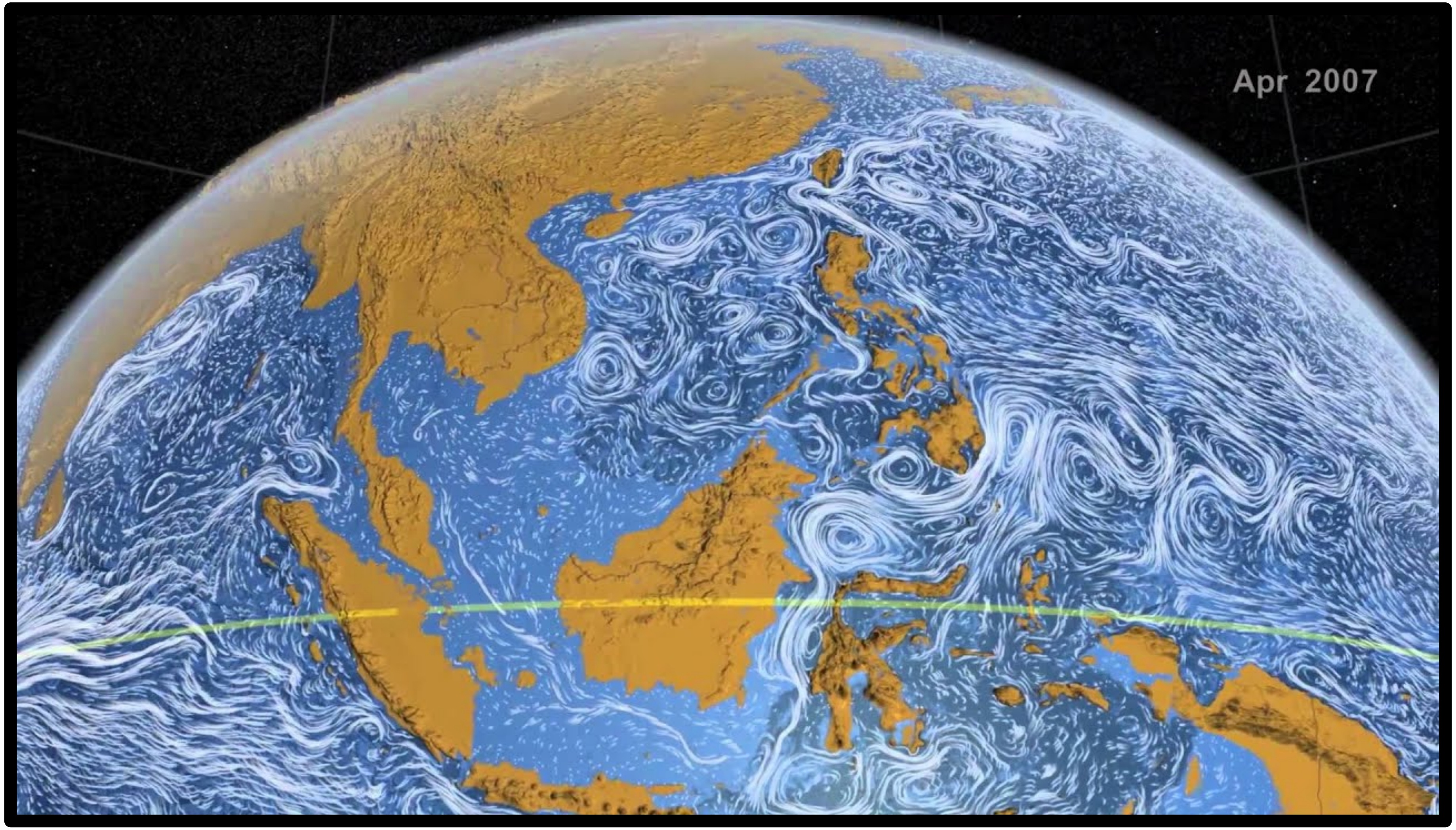


The Earth System: Ocean Dynamics



US DOE E3SM (left) Simulated ocean current speed and (right) concurrent ocean temperature

The Earth System: Ocean Dynamics



NASA Perpetual Ocean https://www.youtube.com/watch?v=CCmTY0PKGDs

Ocean Eddies

Ocean drifter data, can be analyzed and decomposed into a time-averaged component and time-dependent component

$$\eta(x,y,t) = \overline{\eta}(x,y) + \eta'(x,y,t)$$

The standard deviation of the surface height about the time mean is then given by

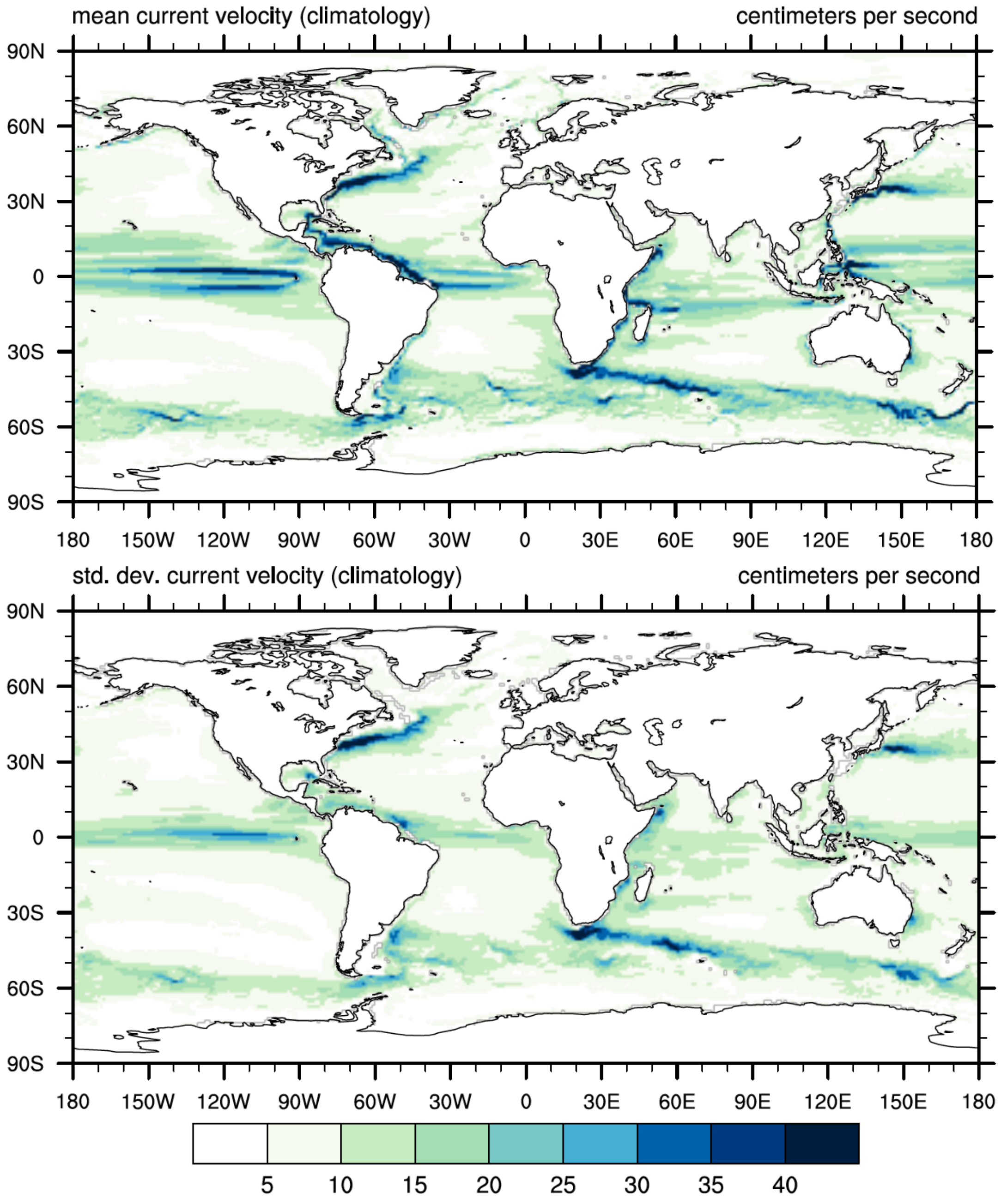
$$\sigma_\eta = \sqrt{\overline{\eta'^2}}$$

Ocean Eddies

Figure: Mean and standard deviation of surface current speeds from SODA3.

Regions in which observations are sparse, particularly in the southern oceans around Antarctica, appear as white gaps.

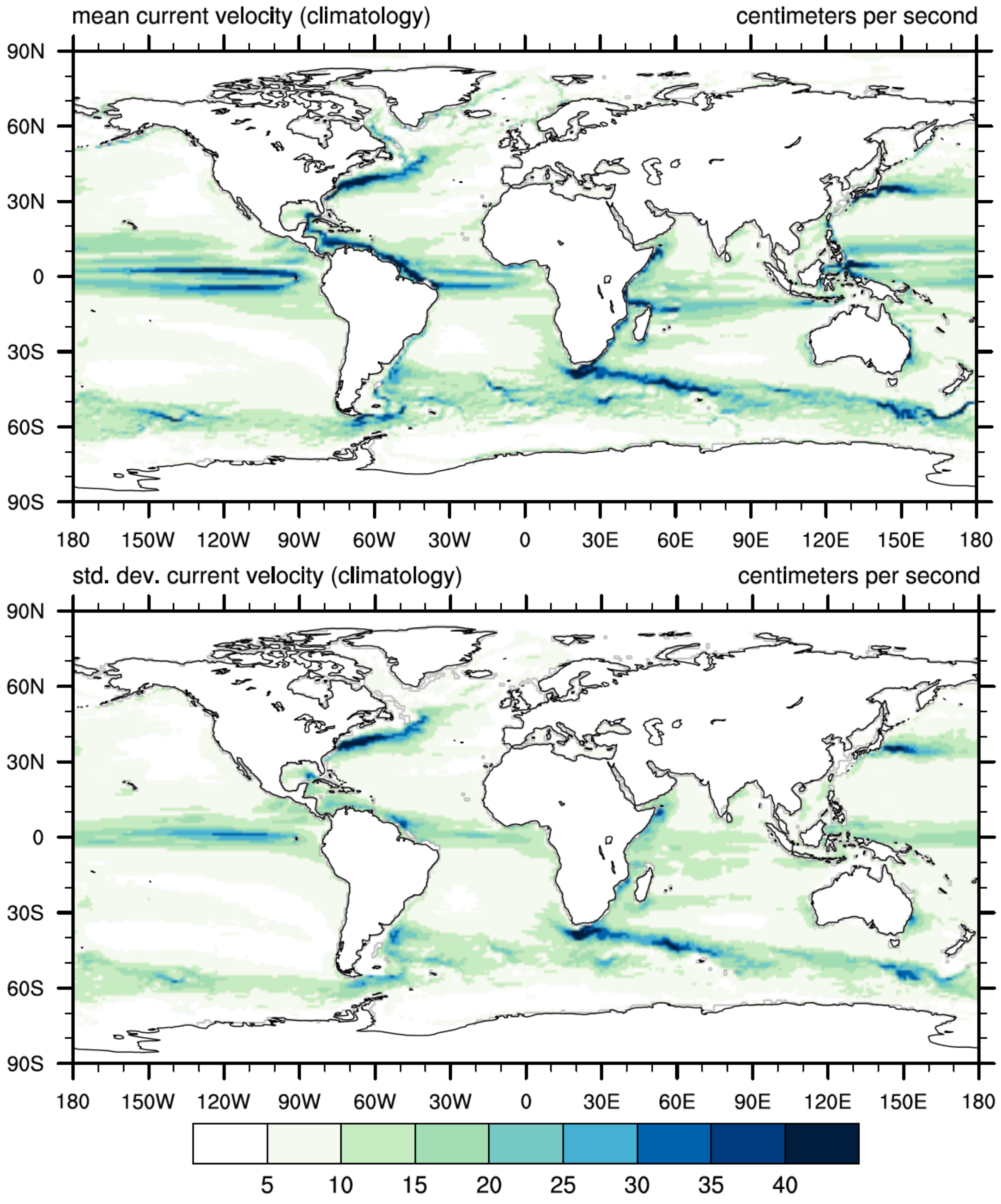
Observe that the deviation from the mean is of the same order of magnitude as the mean, so eddies can lead to backwards propagation relative to the mean current.



Ocean Eddies

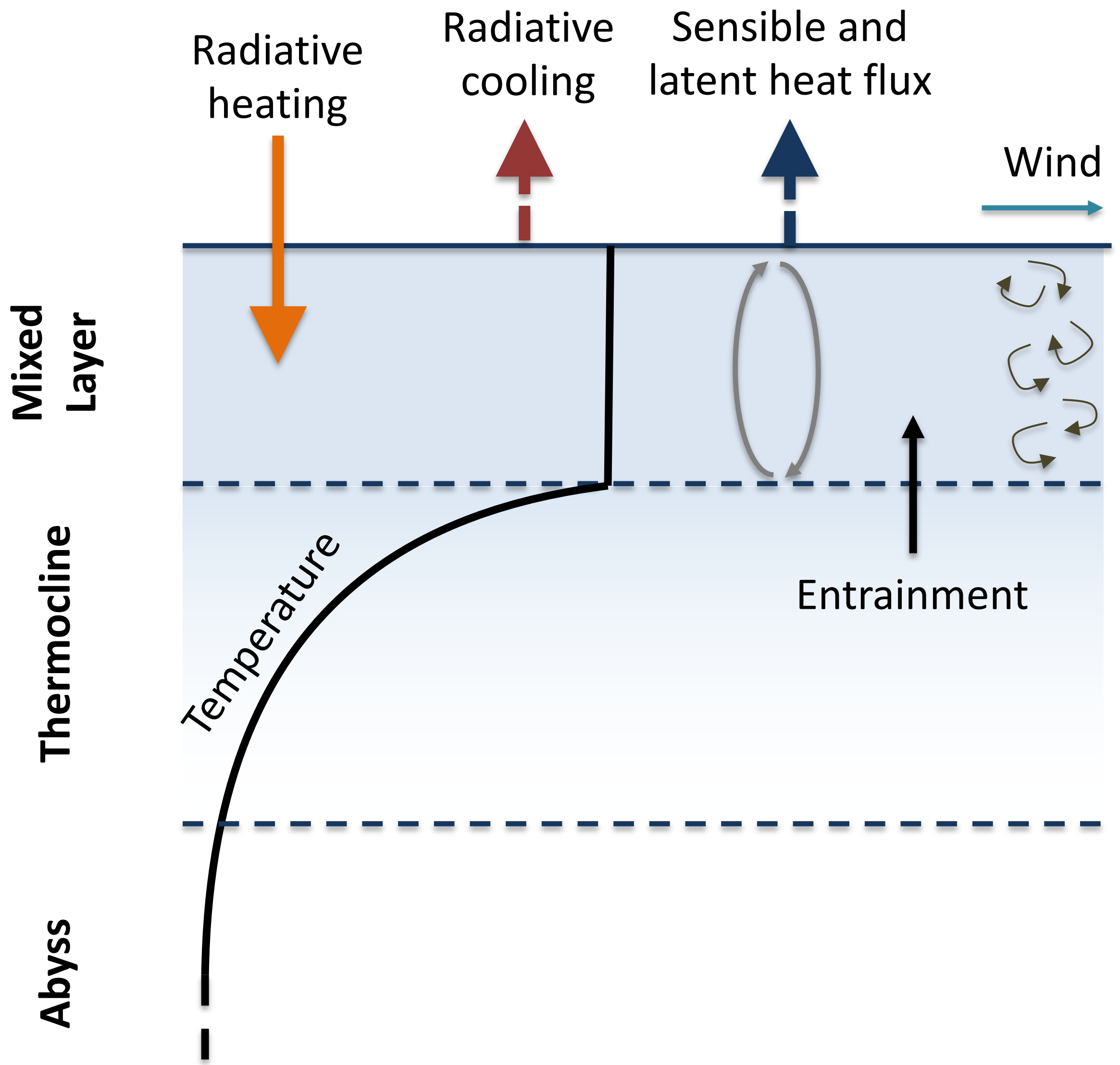
These figures lead us to the conclusion that the ocean is not steady and laminar, but is highly turbulent.

Oceanic eddies (~100km with lifetimes of months) are much different than their atmospheric counterparts (~1000km with lifetimes of days).



Wind Stress and Ekman Layers

Ocean Temperature Structure



Definition: The **Mixed Layer** is a layer in which active turbulence has homogenized some range of depths.

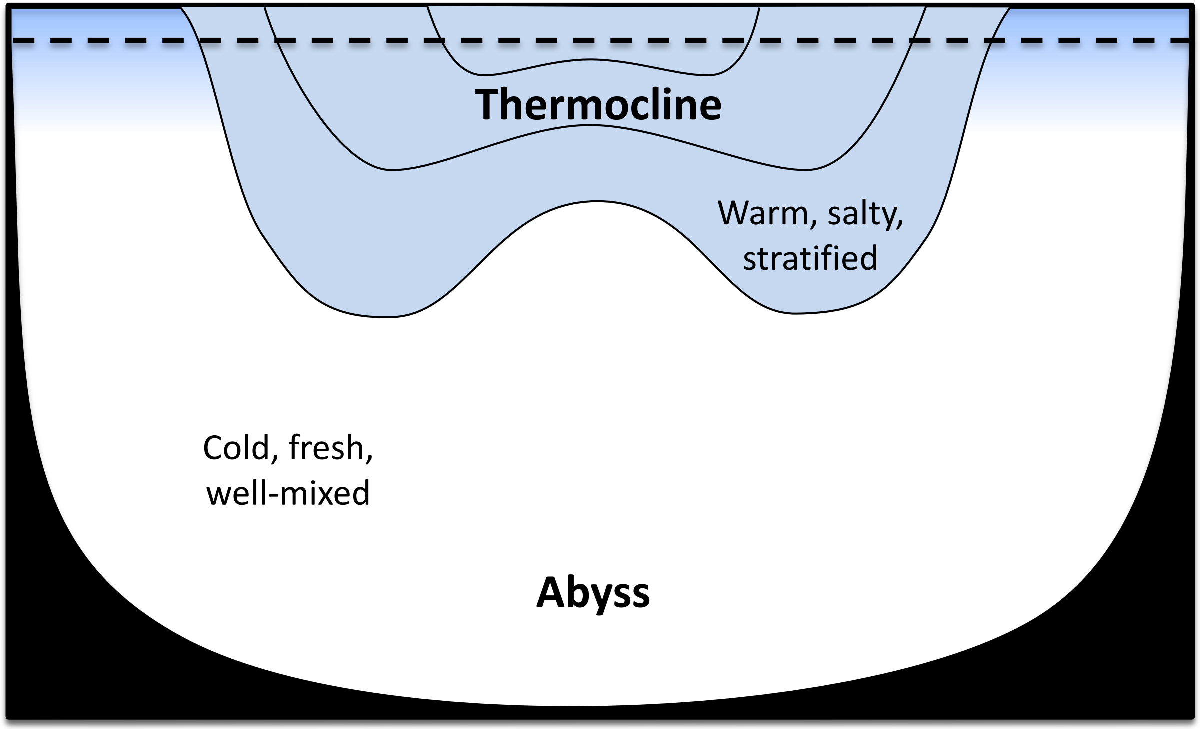
Definition: The **Thermocline** is a layer of fluid where the temperature changes more rapidly than in layers above or below.

Definition: The **Abyss** is the deepest part of the open ocean, below 4000m depth. This region is in complete darkness and (largely) thermodynamically homogeneous.

Vertical-Meridional Structure

To summarize, the vertical structure of the ocean consists of:

- A warm, salty, stratified lens of fluid near the surface.
- A rapid drop in temperature separating the surface layer from the interior ocean (the thermocline).
- A relatively well-mixed abyss, with very weak mean flow.



Vertical-Meridional Circulation

In a simplified sense, the ocean circulation is driven by the wind-driven circulation and the thermohaline circulation.

Definition: The **wind-driven circulation** refers to tangential stresses at the ocean's surface due to the prevailing wind systems.

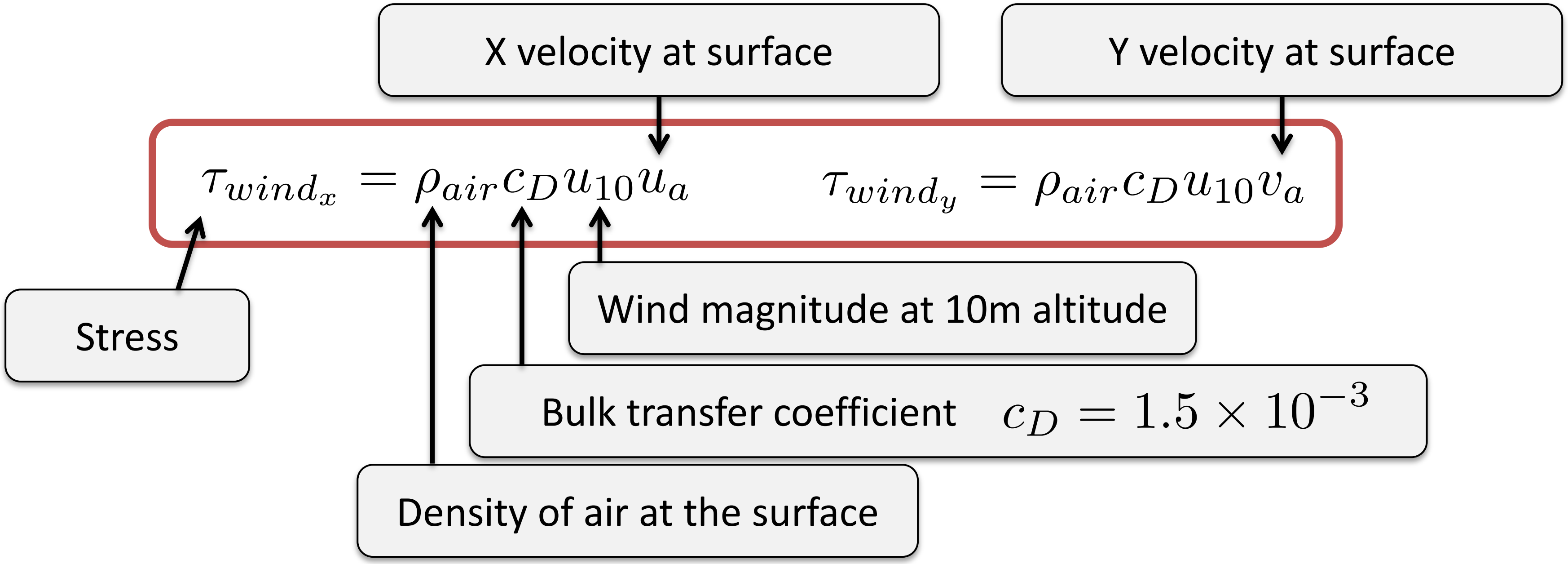
Definition: The **thermohaline circulation** refers to a global circulation driven by convection at polar latitudes.

Geostrophic currents (which we studied last time) are driven by horizontal gradients in the density field which are induced by Ekman pumping / suction (in turn induced by the wind-driven circulation).

Surface Wind Stress

Definition: The **surface wind stress** is the force on the ocean surface induced by friction at the atmosphere-ocean interface.

Wind stress is related to the wind velocity through the "bulk formula", which connects gradients in wind speed and induced forces.



X velocity at surface

Y velocity at surface

$$\tau_{wind_x} = \rho_{air} c_D u_{10} u_a \qquad \tau_{wind_y} = \rho_{air} c_D u_{10} v_a$$

Stress

Wind magnitude at 10m altitude

Bulk transfer coefficient $c_D = 1.5 \times 10^{-3}$

Density of air at the surface

Surface Wind Stress

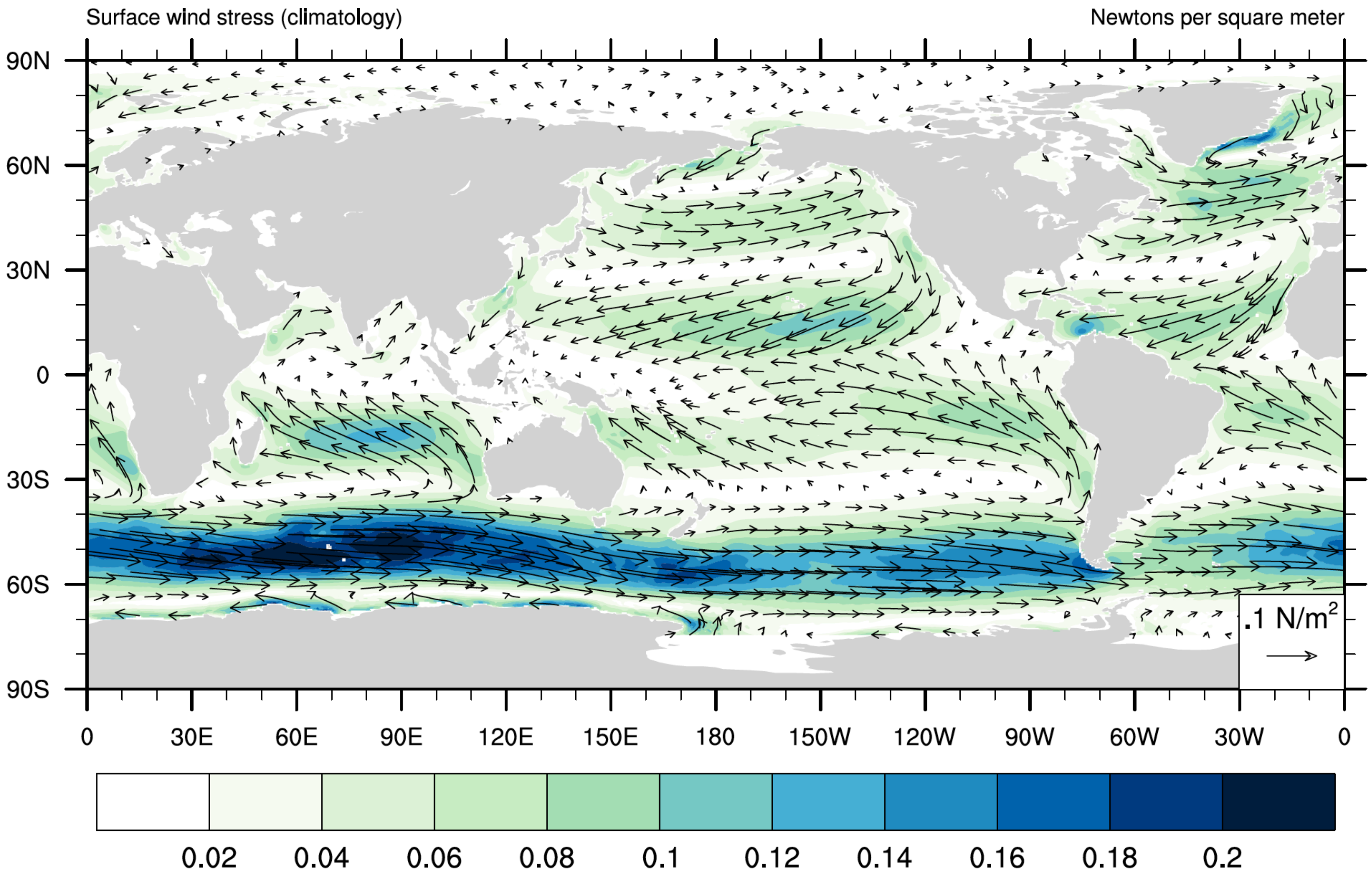
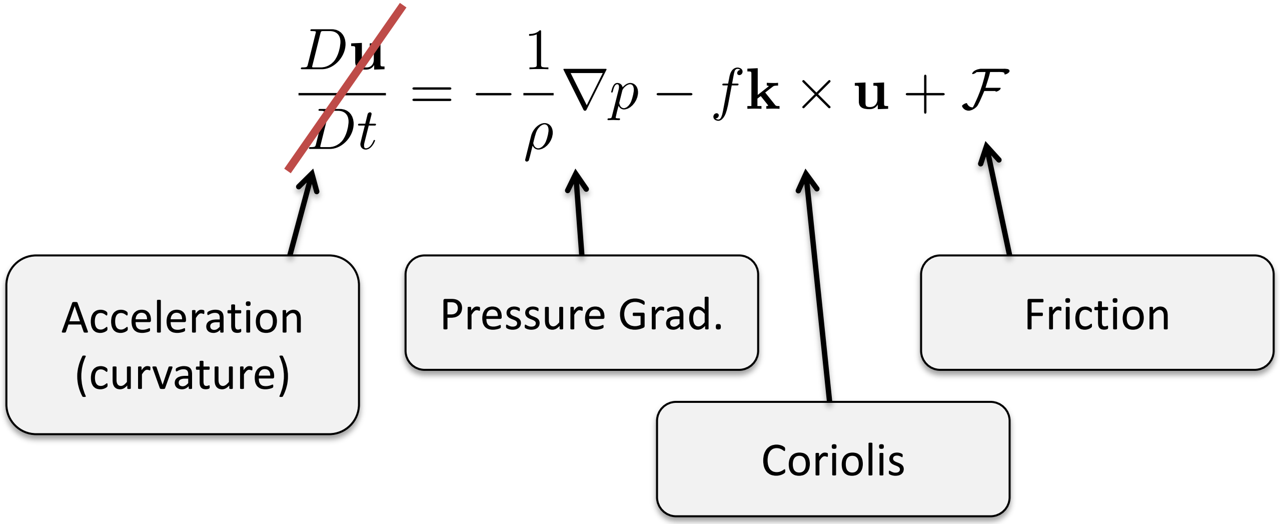


Figure: Surface wind stress from the atmosphere on the surface ocean (N/m^2).

The Oceanic Ekman Layer

Recall the derivation of subgeostrophic flow:



$$\cancel{\frac{D\mathbf{u}}{Dt}} = -\frac{1}{\rho}\nabla p - f\mathbf{k}\times\mathbf{u} + \mathcal{F}$$

Acceleration (curvature)

Pressure Grad.

Coriolis

Friction

Question: What is friction?

Answer: An applied body force due to the action of the wind on the ocean.

The Oceanic Ekman Layer

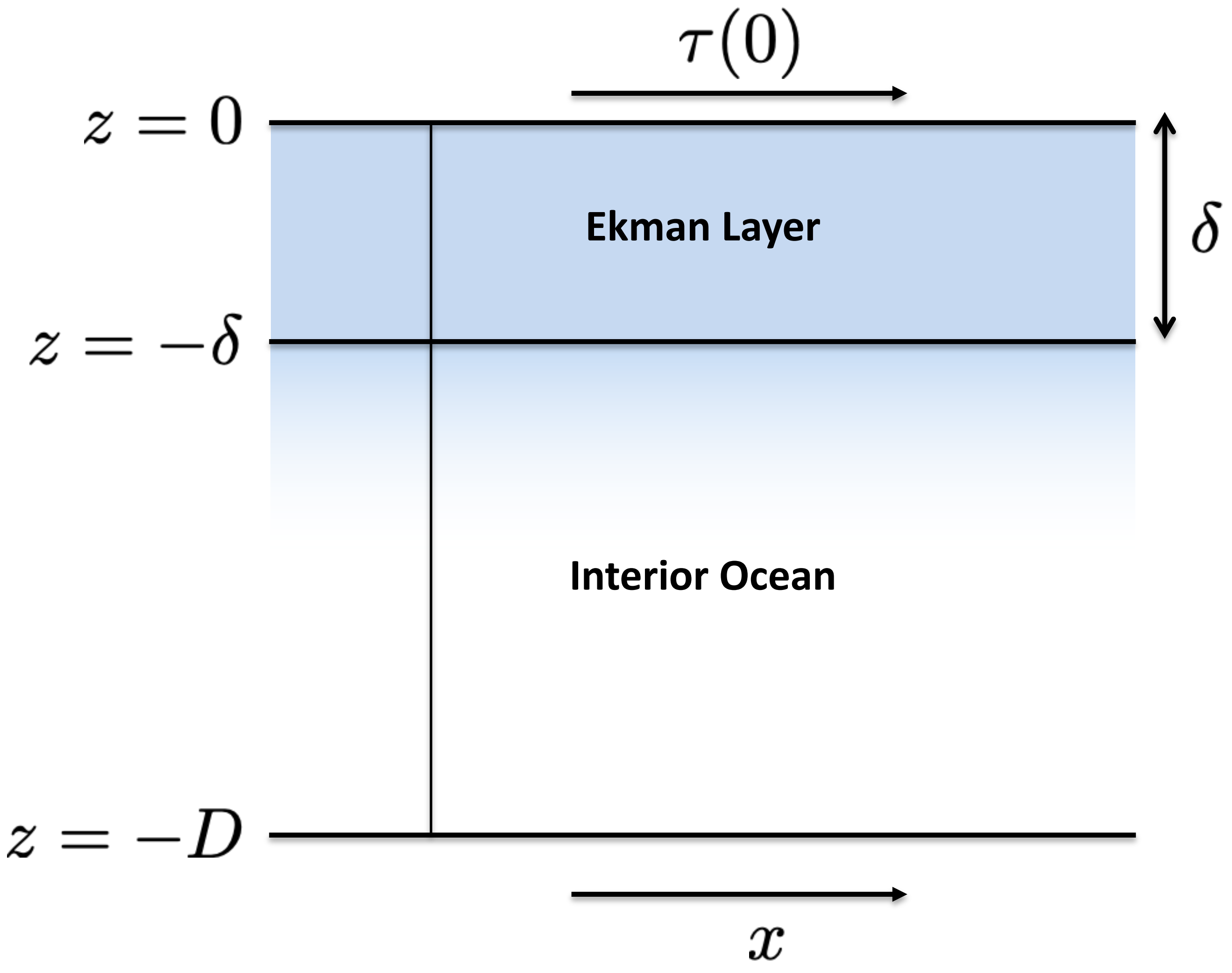


Figure: The stress applied to an elemental slab of fluid of depth δz = δ diminishes with depth.

Hence,

$$\mathcal{F}_x = \frac{\text{force per unit area}}{\text{mass per unit area}}$$

$$= \frac{\tau_x(z+\delta z) - \tau_x(z)}{\rho_{\text{ref}}\delta z} = \frac{1}{\rho_{\text{ref}}}\frac{\partial \tau_x}{\partial z}$$

The Oceanic Ekman Layer

Balance law within the Ekman layer:

$$-fv + \frac{1}{\rho_{\text{ref}}}\frac{\partial p}{\partial x} = \frac{1}{\rho_{\text{ref}}}\frac{\partial \tau_x}{\partial z}$$
$$fu + \frac{1}{\rho_{\text{ref}}}\frac{\partial p}{\partial y} = \frac{1}{\rho_{\text{ref}}}\frac{\partial \tau_y}{\partial z}$$

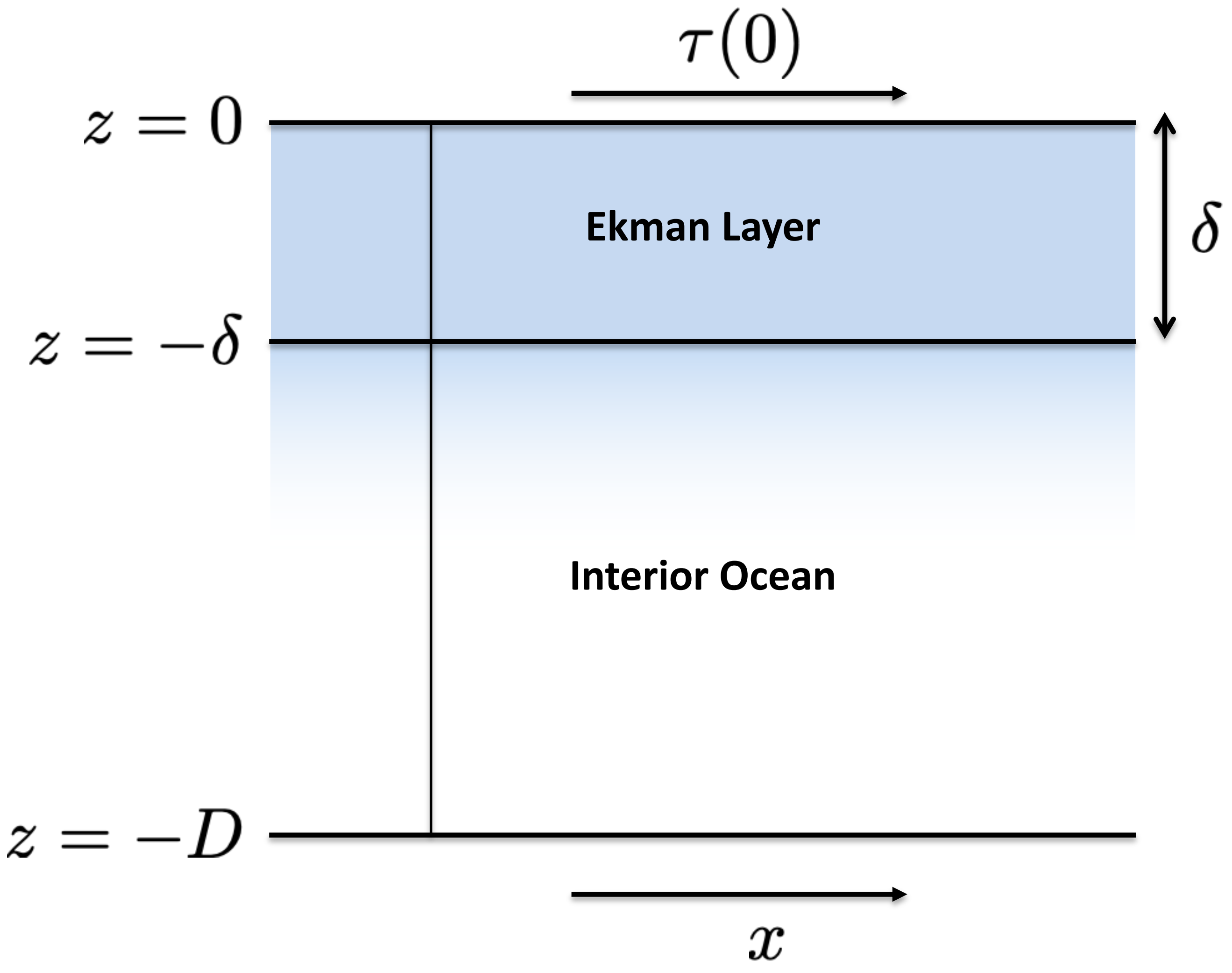
To solve for the circulation, we need to know the distribution of wind stress through the Ekman layer. The stress at the surface is known:

$$\tau(0) = \tau_{wind}$$

From observations, the direct influence of wind forcing decays with depth rather rapidly (in a few tens of meters) so that at z = -δ the stress is zero:

$$\tau(-\delta) = 0$$

The Oceanic Ekman Layer



Typical values of δ are

$$\delta \approx 10 - 100 \text{ m}$$

Like the atmosphere, the region of the ocean’s surface where wind stress and frictional effects are important is known as the **Ekman layer**.

The Oceanic Ekman Layer

$$-fv + \frac{1}{\rho_{\mathrm{ref}}}\frac{\partial p}{\partial x} = \frac{1}{\rho_{\mathrm{ref}}}\frac{\partial \tau_x}{\partial z}$$

$$fu + \frac{1}{\rho_{\mathrm{ref}}}\frac{\partial p}{\partial y} = \frac{1}{\rho_{\mathrm{ref}}}\frac{\partial \tau_y}{\partial z}$$

Split wind into geostrophic and ageostrophic parts:

$$\mathbf{u} = \mathbf{u}_g + \mathbf{u}_{ag}$$

Then:

$$f\mathbf{k} \times \mathbf{u}_{ag} = \frac{1}{\rho_{\mathrm{ref}}}\frac{\partial \tau}{\partial z}$$

$$f\mathbf{k} \times \mathbf{u}_{ag} = \frac{1}{\rho_{\mathrm{ref}}}\frac{\partial \tau}{\partial z}$$

Multiply by ρ_{ref} and integrate over the Ekman layer:

$$f\mathbf{k} \times \mathbf{M}_{ek} \approx \tau_{wind}$$

$$\mathbf{M}_{ek} = \int_{-\delta}^{0} \rho_{\mathrm{ref}}\mathbf{u}_{ag}\,dz$$

is the total mass transported by the Ekman layer.

The Oceanic Ekman Layer

$$f\mathbf{k} \times \mathbf{M}_{ek} \approx \tau_{wind}$$

Use the relationship

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{M}_{ek}) = -\mathbf{M}_{ek}$$

And so:

$$\mathbf{M}_{ek} = \frac{\tau_{wind} \times \mathbf{k}}{f}$$

That is, the mass flux is perpendicular to the wind stress. Currents must have a rightward-bias.

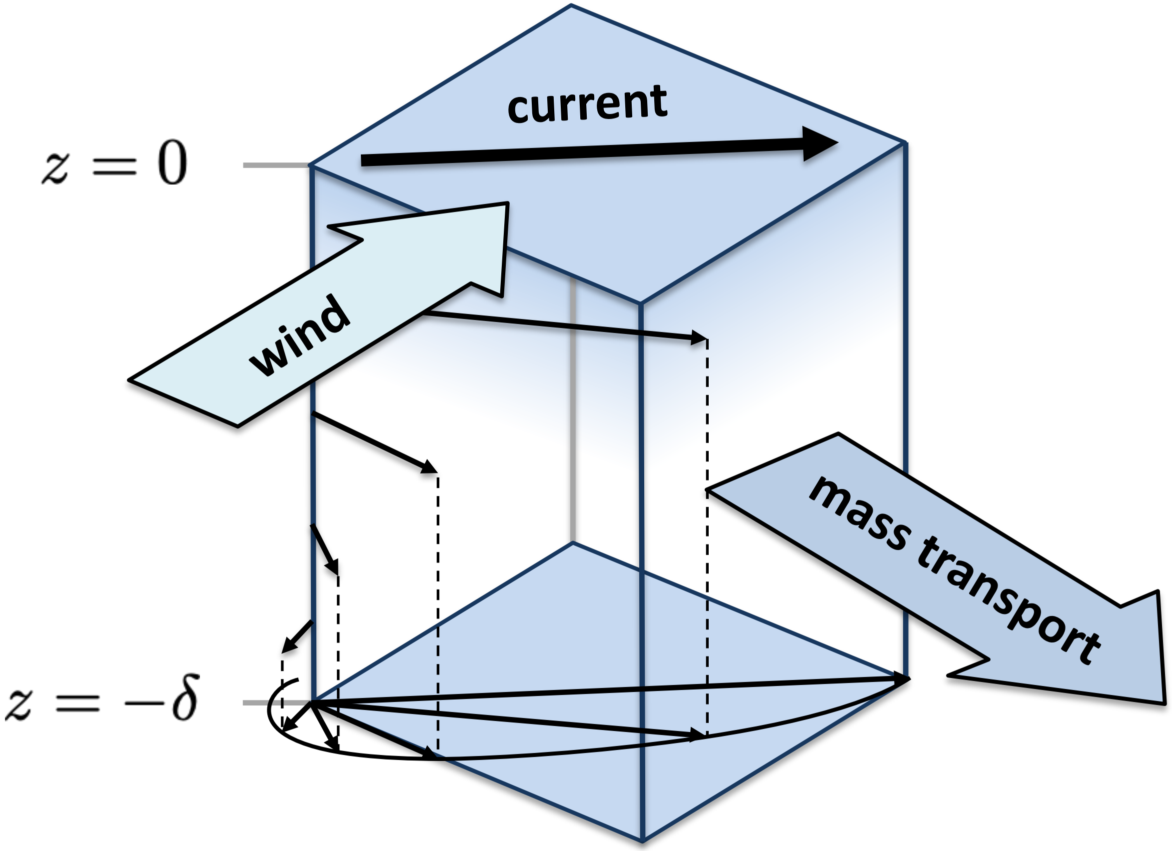
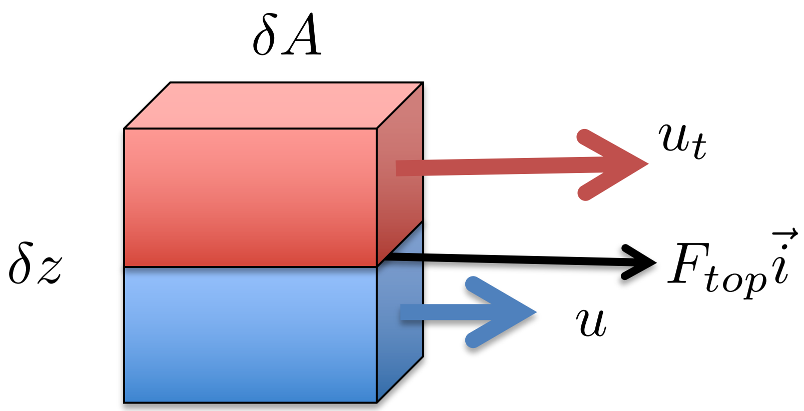


Figure: Mass transport of the Ekman layer is directed to the right of the wind in the Northern hemisphere. Horizontal currents within the Ekman layer spiral are shown.

The Oceanic Ekman Layer

In fact, the stress induced on ocean parcels can be more accurately characterized using the viscous formulation (recall fluid equations lecture):



The shear stress in the x direction due to motion of the red fluid parcel and blue fluid parcel is proportional to the difference in velocities and inversely proportional to the separation:

$$\tau_x \sim \frac{(u_t - u)}{\delta z}$$

This is a common **parameterization** of the surface stresses induced by eddy turbulence.

$$\tau_x \approx \underline{K \rho_{\mathrm{ref}}} \frac{\partial u}{\partial z}$$

Proportionality constant

The Oceanic Ekman Layer

$$\tau_x \approx K \rho_{\text{ref}} \frac{\partial u}{\partial z}$$

$$\mathcal{F}_x = \frac{1}{\rho_{\text{ref}}} \frac{\partial \tau_x}{\partial z}$$

Balance law within the Ekman layer (parameterized):

$$-fv + \frac{1}{\rho_{\text{ref}}} \frac{\partial p}{\partial x} - K \frac{\partial^2 u}{\partial z^2} = 0$$

$$fu + \frac{1}{\rho_{\text{ref}}} \frac{\partial p}{\partial y} - K \frac{\partial^2 v}{\partial z^2} = 0$$

Using the geostrophic wind relationship:

$$fu_g = -\frac{1}{\rho_{\text{ref}}} \frac{\partial p}{\partial y} \qquad fv_g = \frac{1}{\rho_{\text{ref}}} \frac{\partial p}{\partial x}$$

We have:

$$f(v_g - v) - K \frac{\partial^2 u}{\partial z^2} = 0$$

$$-f(u_g - u) - K \frac{\partial^2 v}{\partial z^2} = 0$$

The Oceanic Ekman Layer

$$\begin{aligned} f(v_g - v) - K\frac{\partial^2 u}{\partial z^2} &= 0 \\ -f(u_g - u) - K\frac{\partial^2 v}{\partial z^2} &= 0 \end{aligned}$$

Assume the geostrophic velocity doesn’t vary much in the Ekman layer. Then solve for the ageostrophic wind:

$$\begin{aligned} -fv_{ag} - K\frac{\partial^2 u_{ag}}{\partial z^2} &= 0 \\ fu_{ag} - K\frac{\partial^2 v_{ag}}{\partial z^2} &= 0 \end{aligned}$$

The Oceanic Ekman Layer

$$-fv_{ag} - K\frac{\partial^2 u_{ag}}{\partial z^2} = 0$$
$$fu_{ag} - K\frac{\partial^2 v_{ag}}{\partial z^2} = 0$$

Two coupled second-order constant coefficient partial differential equations (four solutions)

Boundary conditions:

(1) at $z = 0$ $\quad \tau(0) = \tau_{wind}$ $\quad \Rightarrow \quad \frac{\partial \mathbf{u}_{ag}}{\partial z} = \frac{\tau_{wind}}{K\rho_{\text{ref}}}$

(2) as $z \to -\infty$ $\quad \mathbf{u}_{ag} \to 0$

No growing solutions allowed

Boundary condition applies to stress (not a no-slip condition)

The Oceanic Ekman Layer

$$\boxed{\begin{aligned} -fv_{ag} - K\frac{\partial^2 u_{ag}}{\partial z^2} &= 0 \\ fu_{ag} - K\frac{\partial^2 v_{ag}}{\partial z^2} &= 0 \end{aligned}}$$

(1) at $z = 0$ $\quad \dfrac{\partial \mathbf{u}_{ag}}{\partial z} = \dfrac{\tau_{wind}}{K\rho_{\text{ref}}}$

(2) as $z \to -\infty$ $\quad \mathbf{u}_{ag} \to 0$

Solution for $\dfrac{\tau_{wind,x}}{K\rho_{\text{ref}}} = \dfrac{U}{\delta} \quad \tau_{wind,y} = 0$

Purely zonal wind stress

$$u_{ag}(z) = \frac{U}{2\delta\gamma}\left[\cos\gamma z + \sin\gamma z\right] e^{\gamma z}$$

$$v_{ag}(z) = -\frac{U}{2\delta\gamma}\left[-\sin\gamma z + \cos\gamma z\right] e^{\gamma z} \cdot \text{sign}(f)$$

where $\gamma = \sqrt{|f|/2K}$

The Oceanic Ekman Layer

Solution for

$$\frac{\tau_{wind,x}}{K\rho_{\mathrm{ref}}} = \frac{U}{\delta} \qquad \tau_{wind,y} = 0$$

at the surface:

$$u_{ag}(0) = \frac{U}{2\delta\gamma} \qquad v_{ag}(0) = -\frac{U}{2\delta\gamma}\mathrm{sign}(f)$$

$$\gamma = \sqrt{|f|/2K}$$

Surface currents are directed 45° to the right of the wind stress in the northern hemisphere. Similarly in the southern hemisphere except to the left.

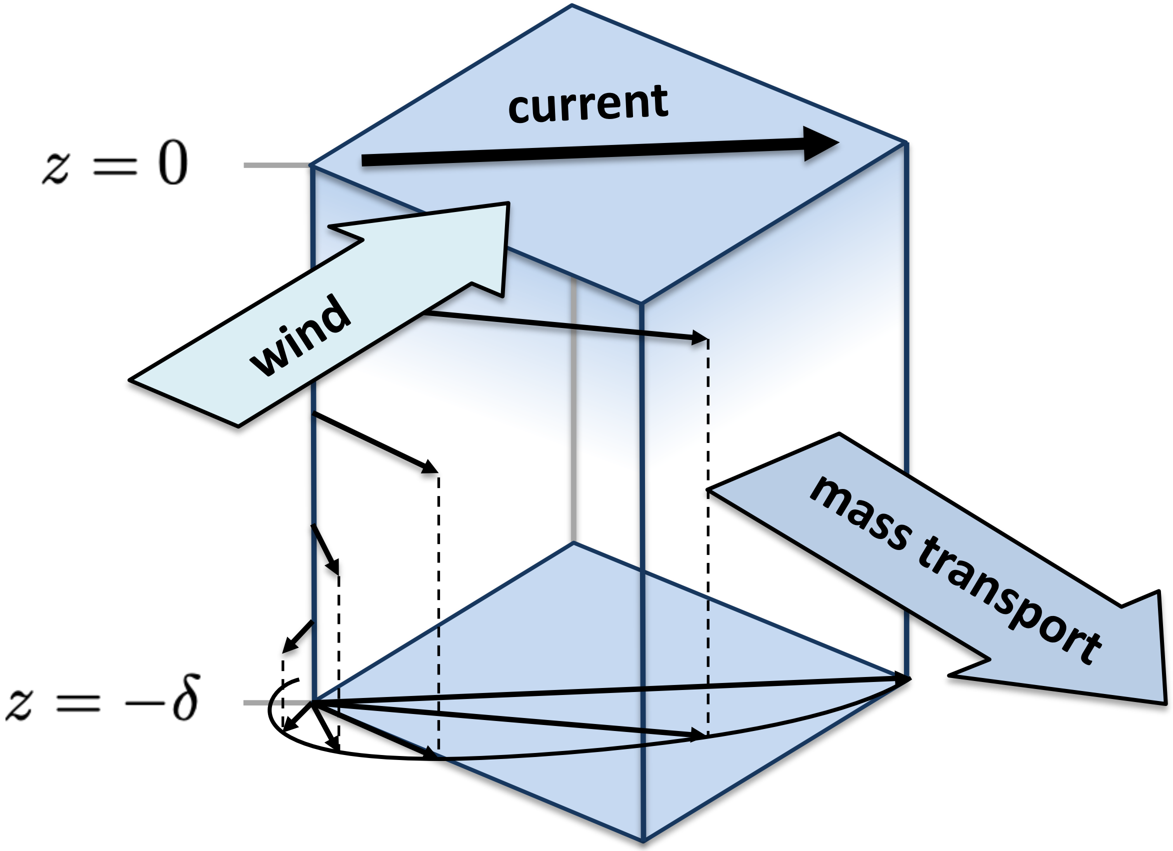


Figure: Mass transport of the Ekman layer is directed to the right of the wind in the Northern hemisphere. Horizontal currents within the Ekman layer spiral are shown.

Surface Wind Stress

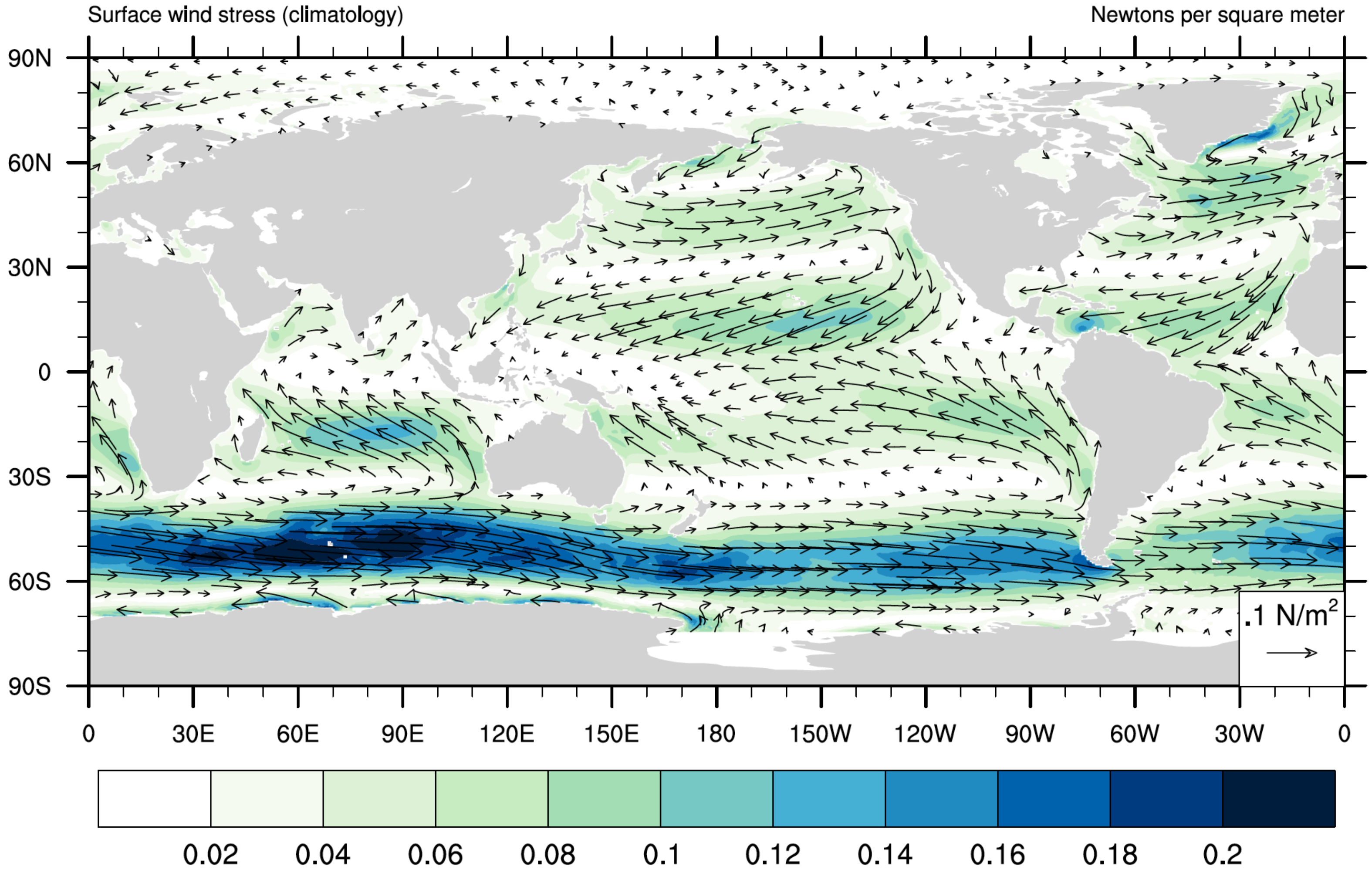


Figure: Surface wind stress from the atmosphere on the surface ocean (N/m^2).

ATM 241 Climate Dynamics
Lecture 9a
The Wind-Driven Circulation (Part 1)

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Thank You!