ATM 241, Spring 2020
Lecture 8
Ocean Dynamics

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Marshall & Plumb

Ch. 9

In this section…

Definitions

- Taylor-Proudman Theorem
- Steric effects

Questions

- How does the concept of balanced flow from atmospheric dynamics translate to the ocean?
- How do the thermodynamic properties of the ocean determine its dynamical character?
- What is the effect of temperature and salinity on sea surface height?

Balanced Flow in the Ocean

The Ocean Fluid Equations

$$\frac{du}{dt} - \frac{uv\tan\phi}{r} + \frac{uw}{r} = -\frac{1}{\rho r\cos\phi}\frac{\partial p}{\partial\lambda} + 2\Omega v\sin\phi - 2\Omega w\cos\phi + \nu\nabla^2 u$$

$$\frac{dv}{dt} + \frac{u^2\tan\phi}{r} + \frac{vw}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial\phi} - 2\Omega u\sin\phi + \nu\nabla^2 v$$

$$\frac{dw}{dt} - \frac{u^2+v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g + 2\Omega u\cos\phi + \nu\nabla^2 w$$

$\nabla\cdot\mathbf{u} = 0$ (Incompressible fluid)

$$c_v\frac{dT}{dt} + p\frac{d}{dt}\left(\frac{1}{\rho}\right) = J$$

$$\frac{dq_i}{dt} = S_i$$

Material derivative:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla$$

Seawater Equation of State

Approximated equation of state by Taylor expansion $\sigma = \rho - \rho_{\text{ref}}$

$$\sigma(T,S) \approx \sigma_0 + \rho_{\text{ref}}\Big(-\alpha_T\,[T - T_0] + \beta_S\,[S - S_0]\Big)$$

Thermal expansivity:

$$\alpha_T = -\frac{1}{\rho_{\text{ref}}}\left.\frac{\partial \rho}{\partial T}\right|_{T=T_0,S=S_0}$$

Effect of salinity on density:

$$\beta_S = \frac{1}{\rho_{\text{ref}}}\left.\frac{\partial \rho}{\partial S}\right|_{T=T_0,S=S_0}$$

	Surface			**1km Depth**		
T_0	-1.5°C	5°C	15°C	-1.5°C	3°C	13°C
α (x 10^{-4} K^{-1})	0.3	1	2	0.65	1.1	2.2
S_0 (psu)	34	36	38	34	35	38
β_S (x 10^{-4} psu^{-1})	7.8	7.8	7.6	7.1	7.7	7.4
σ_0 (kg m^{-3})	28	29	28	-3	0.6	6.9

Source: Marshall and Plumb Table 9.4

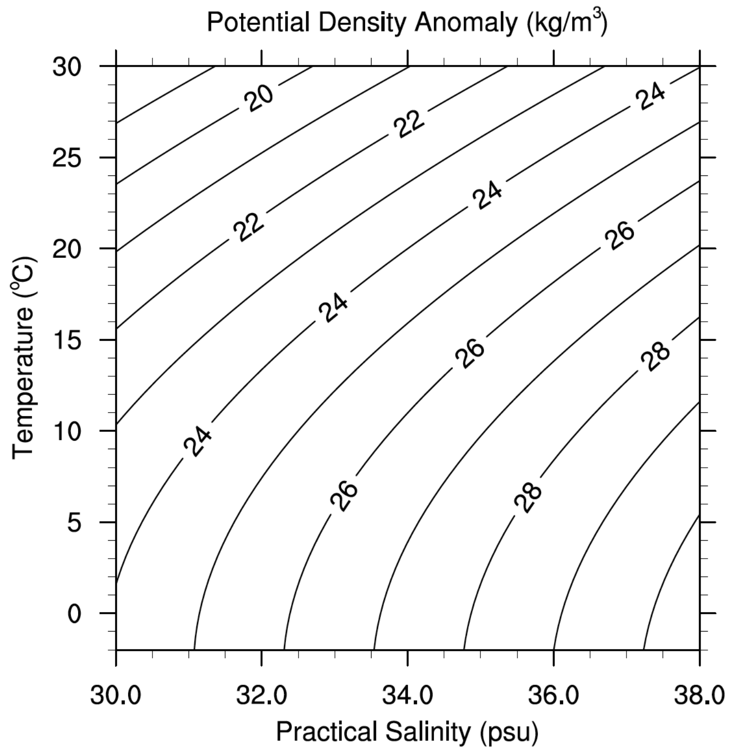


Figure: Density anomalies plotted against salinity and temperature.

Hydrostatic Balance

From the vertical momentum equation:

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g$$

Assuming vertically balanced flow:

$$\frac{\partial p}{\partial z} = -\rho g$$

Writing this in terms of the density anomaly:

$$\boxed{\frac{\partial p}{\partial z} = -g(\rho_{\text{ref}} + \sigma)}$$

Hydrostatic Balance

$$\frac{\partial p}{\partial z} = -g(\rho_{\text{ref}} + \sigma)$$

Neglecting the contribution from σ, this equation can be integrated directly:

$$\int_p^{p_s} dp = -g\rho_{\text{ref}} \int_z^{\eta} dz$$

Where η is the height of the Ocean's free surface, where $p(\eta) = p_s$

$$p(z) = p_s - g\rho_{\text{ref}}(z - \eta)$$

Observe the linear dependency of pressure with height. How does this compare to the atmosphere?

Hydrostatic Balance

$$p(x, y, z) = p_s - g\rho_{\text{ref}}(z - \eta(x, y))$$

The pressure at a depth of 1km in the ocean is thus about 10^7 Pa or 100 times atmospheric pressure.

Note that the the $-g\rho_{\text{ref}}z$ part of this pressure is dynamically inert (does not drive dynamical circulations at constant depth).

Horizontal pressure gradients are generally only associated with:

- Variations in the free surface height $\eta(x, y)$
- Time-mean horizontal variations in surface atmospheric pressure (small)
- Interior density anomalies (associated with T and S variations, neglected above)

Buoyancy in the Ocean

Recall our derivation of the **Brunt-Väisälä Frequency** in the atmosphere:

$$\frac{D^2 \Delta z}{Dt^2} + \frac{g}{T(z_0)}(\Gamma_d - \Gamma)\Delta z = 0$$

$$\frac{g(\Gamma_d - \Gamma)}{T(z_0)} > 0$$

Stable atmosphere

Brunt-Vaisala Frequency

$$\mathcal{N}^2 = \frac{g(\Gamma_d - \Gamma)}{T(z_0)}$$

Units of seconds²

$$\Delta z = \Delta z_0 \sin(\mathcal{N} t + \phi)$$

Oscillatory solutions (magnitude of oscillation depends on initial velocity)

Buoyancy in the Ocean

Figure: Consider a fluid parcel initially located at height z_1 in an environment whose density is $\rho(z)$. The fluid parcel has density $\rho_1 = \rho(z_1)$.

It is now displaced adiabatically a small vertical distance $z_2 = z_1 + \delta z$. By Taylor series:

$$\rho_E = \rho(z_2) \approx \rho_1 + \left(\frac{\partial \rho}{\partial z}\right)_{z=z_1} \delta z$$

$$a_{buoyant} = -g \frac{(\rho_p - \rho_E)}{\rho_p}$$

$$a_{buoyant} = \frac{g}{\rho_1} \left(\frac{\partial \rho}{\partial z}\right)_{z=z_1} \delta z$$

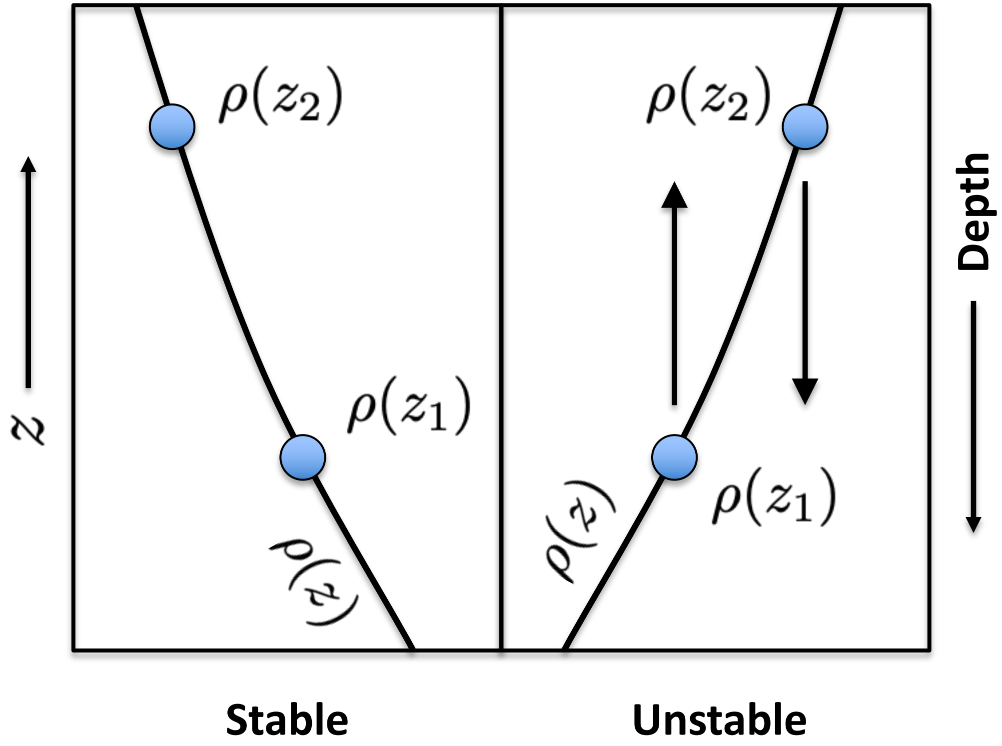


Figure: Convectively stable and unstable vertical density profiles in the ocean.

Buoyancy in the Ocean

How about in the ocean?

$$\frac{D^2 \Delta z}{Dt^2} = \frac{g}{\rho_E} \frac{\partial \rho_E}{\partial z} \Delta z$$

Stable ocean $\frac{g}{\rho_E} \frac{\partial \rho_E}{\partial z} < 0$

$$\rho_E = \rho_{\text{ref}} + \sigma$$

Brunt-Vaisala Frequency

$$\mathcal{N}^2 = -\frac{g}{\rho_E} \frac{\partial \sigma}{\partial z}$$

Units of seconds $^{-2}$

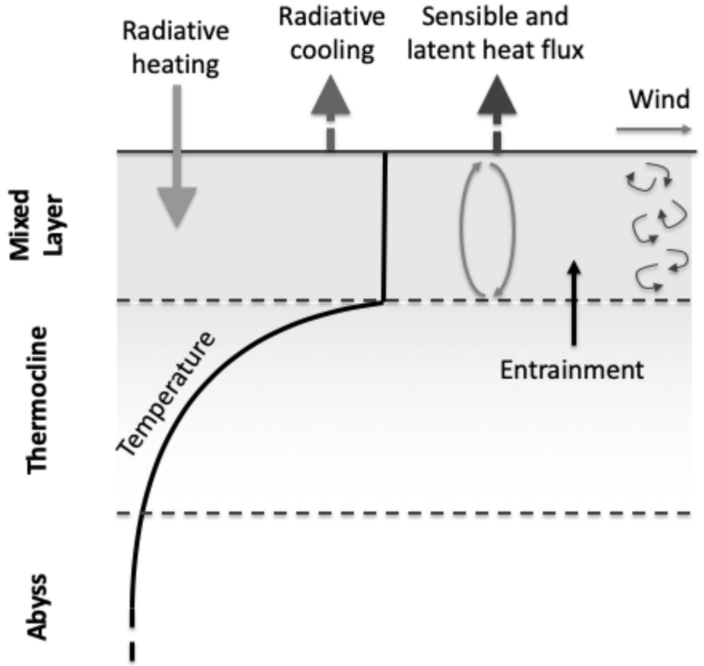
$$\Delta z = \Delta z_0 \sin(\mathcal{N} t + \phi)$$

Oscillatory solutions (magnitude of oscillation depends on initial velocity)

Buoyancy in the Ocean

Brunt-Vaisala Frequency

$$\mathcal{N}^2 = -\frac{g}{\rho_E}\frac{\partial \sigma}{\partial z}$$



Near the thermocline temperature gradients are much larger than salinity gradients, and so

$$\mathcal{N}^2 \approx g\alpha_T \frac{\partial T}{\partial z}$$

α_T Thermal expansion coefficient of water

In surface layer $\alpha_T \approx 2 \times 10^{-4}\ \mathrm{K}^{-1}$

Using $\Delta T \approx 15^\circ\mathrm{C}$

$\Delta z \approx 1000\ \mathrm{m}$

$$\mathcal{N} \approx 5 \times 10^{-3}\ \mathrm{s}^{-1}$$

Or about 20 min. period of oscillation

Geostrophic Balance

Recall the horizontal momentum equation:

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p - f\mathbf{k}\times\mathbf{u} + \mathcal{F}$$

Acceleration (curvature)

Pressure Grad.

Coriolis

Friction

In the ocean, the length scale is determined by the size of typical ocean gyres (about 2000 km). Hence, the Rossby number is

$$Ro = \frac{U}{fL} = \frac{(0.1\ \mathrm{m/s})}{(10^{-4}\ \mathrm{s}^{-1})(2\times 10^{6}\ \mathrm{m})} \sim 10^{-3}$$

Compare: The atmosphere has a Rossby number of 0.1

Geostrophic Balance

For small Rossby number, curvature of the flow can be neglected:

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p - f\mathbf{k} \times \mathbf{u} + \mathcal{F}$$

Acceleration (curvature)

Pressure Grad.

Coriolis

Friction

This implies that geostrophic balance is an excellent approximation for the interior of the ocean **away from the equator** (where Coriolis force is large) and **away from the surface, bottom and lateral boundaries** (where friction is large).

Geostrophic Balance

In the ocean the geostrophic wind is given by

$$u_g = -\frac{1}{f\rho_{\mathrm{ref}}}\left(\frac{\partial p}{\partial y}\right)_z \qquad v_g = \frac{1}{f\rho_{\mathrm{ref}}}\left(\frac{\partial p}{\partial x}\right)_z$$

Since density varies little throughout the ocean, one can replace ρ with ρ_{ref} in the denominator.

Thermal "Wind" in the Ocean

In the ocean the geostrophic wind is given by

$$u_g = -\frac{1}{f\rho_{\mathrm{ref}}}\left(\frac{\partial p}{\partial y}\right)_z \qquad v_g = \frac{1}{f\rho_{\mathrm{ref}}}\left(\frac{\partial p}{\partial x}\right)_z$$

Differentiate with respect to z (approximate f as constant):

$$\frac{\partial u_g}{\partial z} = -\frac{1}{f\rho_{\mathrm{ref}}}\frac{\partial}{\partial y}\frac{\partial p}{\partial z} \qquad \frac{\partial v_g}{\partial z} = \frac{1}{f\rho_{\mathrm{ref}}}\frac{\partial}{\partial x}\frac{\partial p}{\partial z}$$

Use hydrostatic relationship $$\frac{\partial p}{\partial z} = -g(\rho_{\mathrm{ref}} + \sigma)$$

Thermal wind relationship:

$$\frac{\partial u_g}{\partial z} = \frac{g}{f\rho_{\mathrm{ref}}}\frac{\partial \sigma}{\partial y} \qquad \frac{\partial v_g}{\partial z} = -\frac{g}{f\rho_{\mathrm{ref}}}\frac{\partial \sigma}{\partial x}$$

Thermal "Wind" in the Ocean

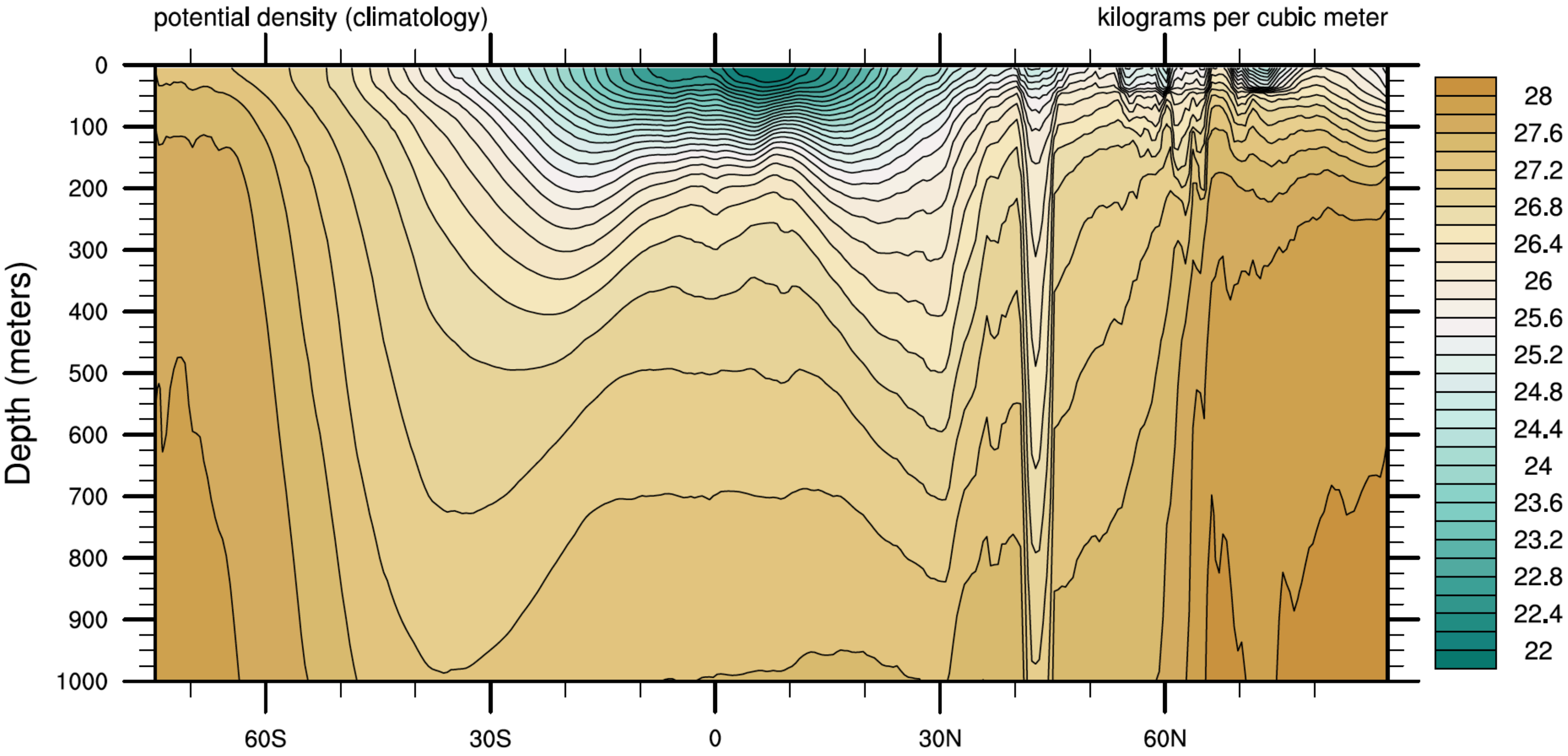
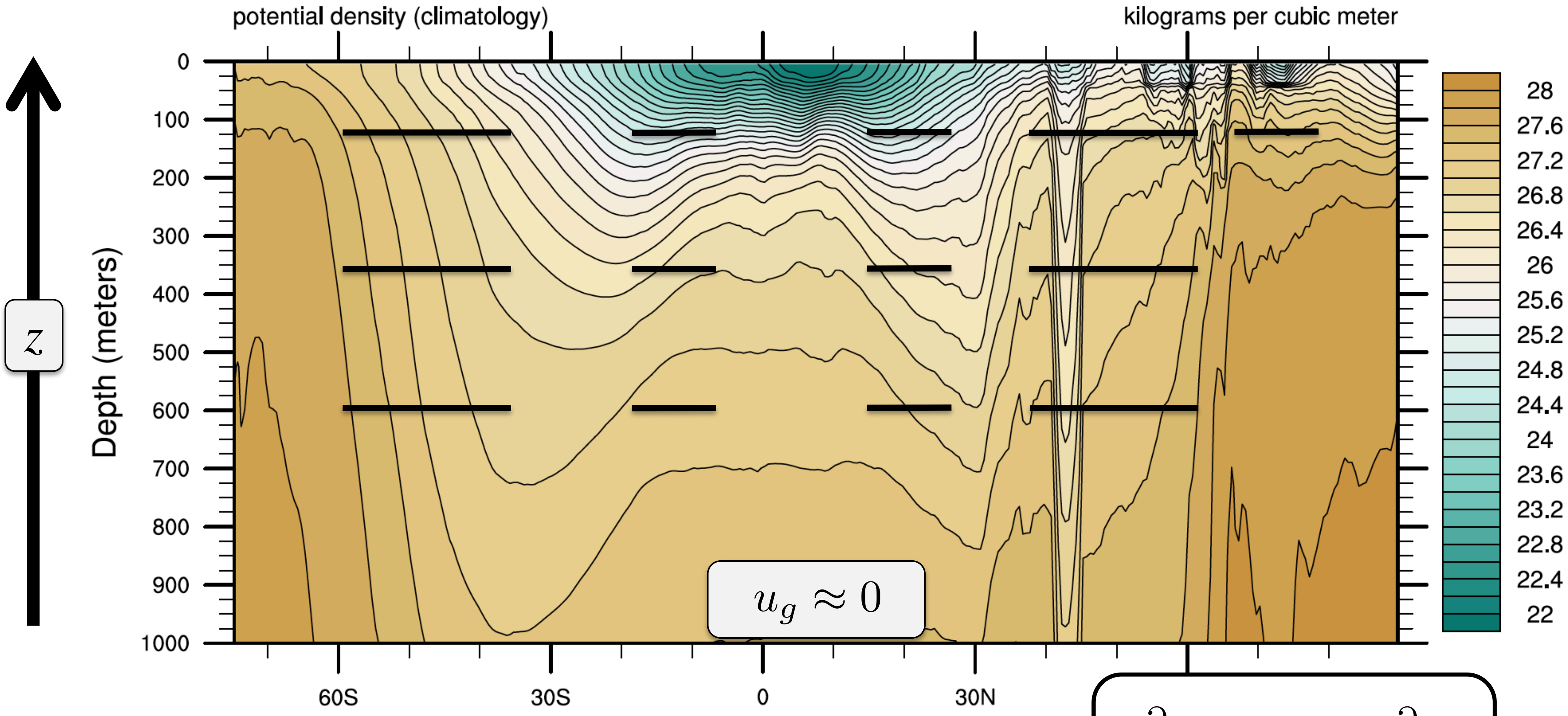


Figure: Annual-mean cross section of zonal average potential density σ.

Thermal "Wind" in the Ocean

$\frac{\partial \sigma}{\partial y}$?



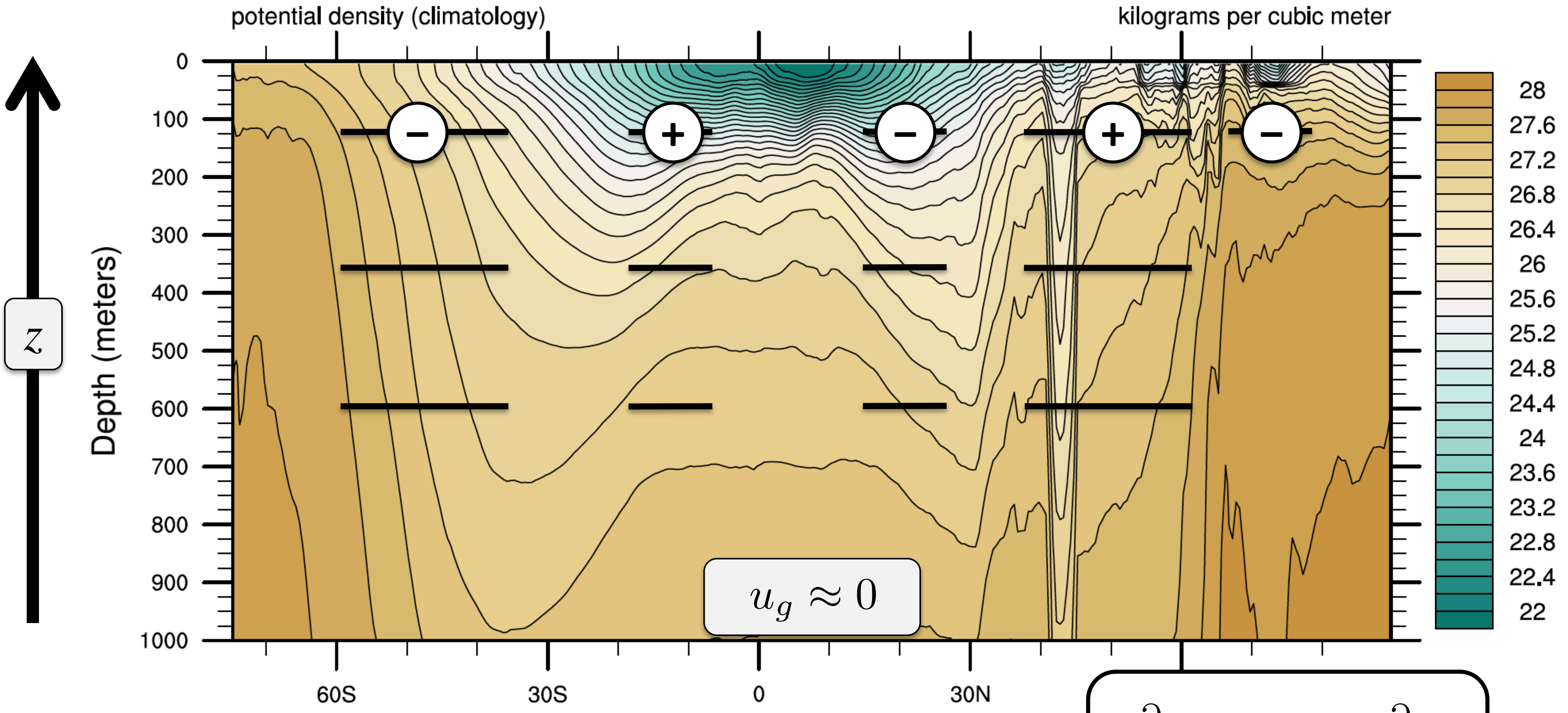
$u_g \approx 0$

Question: Where are the major east-west oceanic currents?

$$\frac{\partial u_g}{\partial z} = \frac{g}{f \rho_{\text{ref}}} \frac{\partial \sigma}{\partial y}$$

Thermal "Wind" in the Ocean

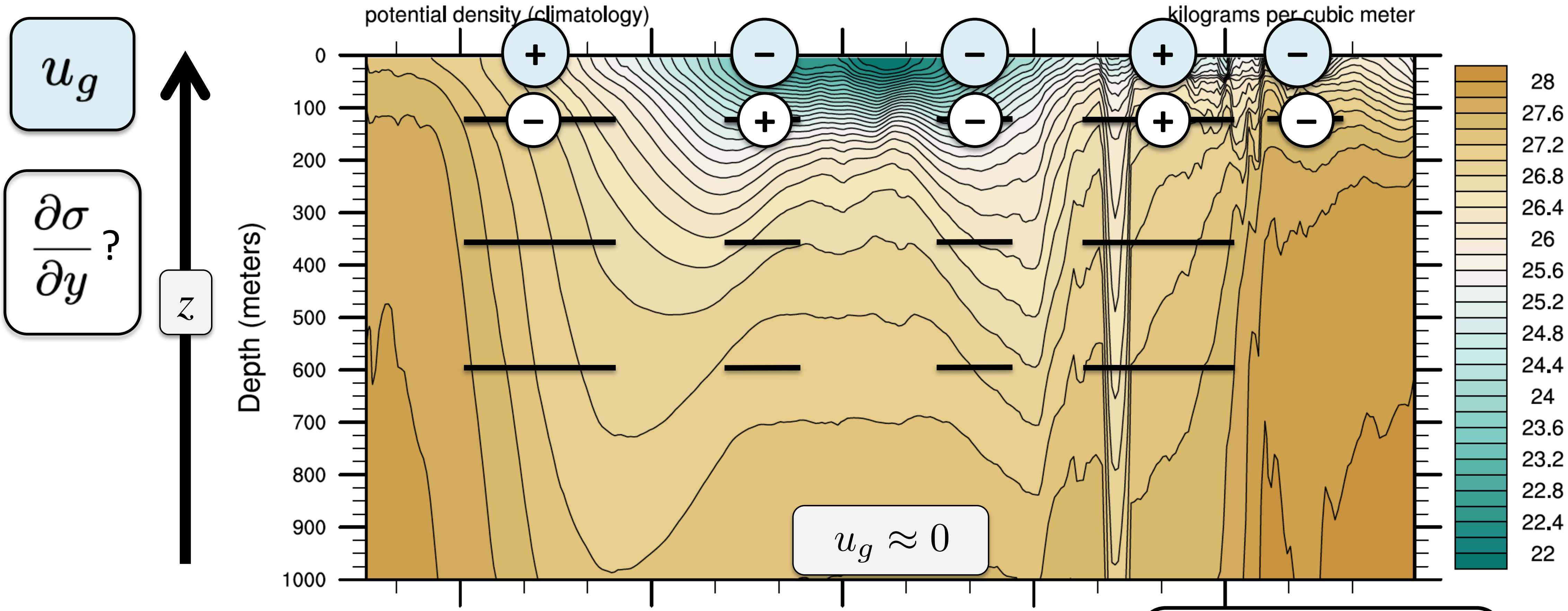
$$\frac{\partial \sigma}{\partial y}\ ?$$



Question: Where are the major east-west oceanic currents?

$$\frac{\partial u_g}{\partial z} = \frac{g}{f \rho_{\text{ref}}} \frac{\partial \sigma}{\partial y}$$

Thermal “Wind” in the Ocean



Question: Where are the major east-west oceanic currents?

$$\frac{\partial u_g}{\partial z} = \frac{g}{f \rho_{\text{ref}}} \frac{\partial \sigma}{\partial y}$$

Global Ocean Currents

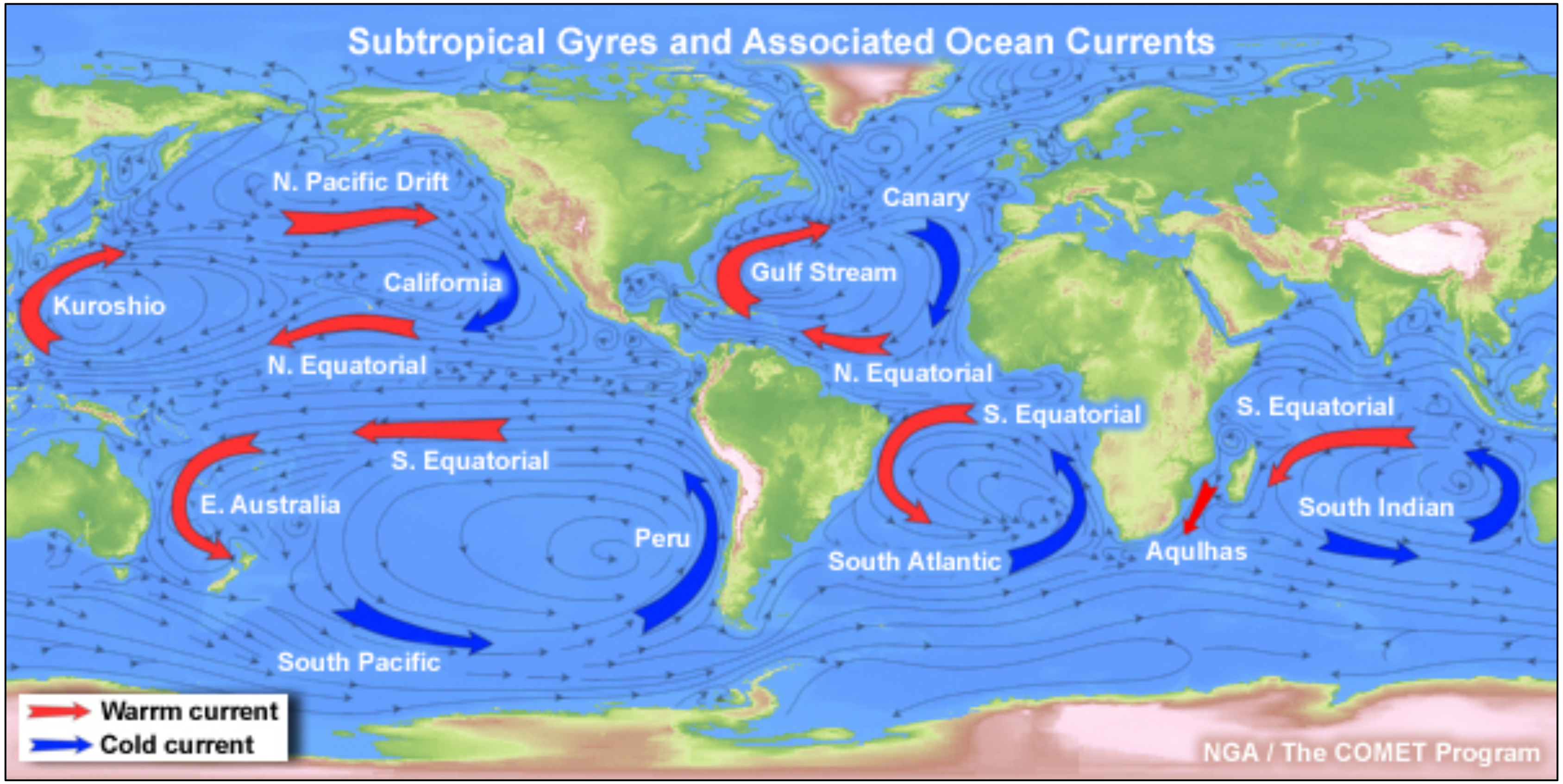


Figure: Subtropical gyres and associated ocean currents (Source: NOAA)
Recall that the direction of shallow ocean currents is largely independent of depth.

Thermal "Wind" in the Ocean

Thermal wind relationship:

$$\frac{\partial u_g}{\partial z} = \frac{g}{f \rho_{\text{ref}}} \frac{\partial \sigma}{\partial y} \qquad \frac{\partial v_g}{\partial z} = -\frac{g}{f \rho_{\text{ref}}} \frac{\partial \sigma}{\partial x}$$

Assuming that the Abyssal currents are weak, the **mean surface geostrophic flow** should be

$$u_{\text{surface}} \sim \frac{g}{f \rho_{\text{ref}}} \frac{H \Delta \sigma}{L} \sim 8 \text{ cm s}^{-1}$$

$\Delta \sigma \sim 1.5 \text{ kg m}^{-3}$ (between 20N and 40N)

$L \sim 2000 \text{ km}$ Horizontal length scale

$H \sim 1000 \text{ m}$ Vertical length scale

Taylor-Proudman Theory

Taylor-Proudman

Taylor-Proudman Theorem:

If flow is geostrophic and homogeneous (ρ = constant) then the flow is two dimensional and does not vary in the direction of the rotation vector.

Proof: From $u_g = -\frac{1}{\rho f}\left(\frac{\partial p}{\partial y}\right)_z \qquad v_g = \frac{1}{\rho f}\left(\frac{\partial p}{\partial x}\right)_z$

Assume ρ and f constant. Then:

$$\frac{\partial u_g}{\partial z} = -\frac{1}{\rho f}\frac{\partial}{\partial y}\frac{\partial p}{\partial z} = -\frac{1}{\rho f}\frac{\partial}{\partial y}(-\rho g) = 0$$

$$\frac{\partial v_g}{\partial z} = \frac{1}{\rho f}\frac{\partial}{\partial x}\frac{\partial p}{\partial z} = \frac{1}{\rho f}\frac{\partial}{\partial x}(-\rho g) = 0$$

So the geostrophic flow does not vary in the direction z.

Taylor-Proudman

Taylor-Proudman Theorem:

If flow is geostrophic and homogeneous (ρ = constant) then the flow is two dimensional and does not vary in the direction of the rotation vector.

Proof (more generally): For low Rossby number flows (low curvature), acceleration can be neglected and so the dynamic primitive equations read

$$2\mathbf{\Omega} \times \mathbf{u} + \frac{1}{\rho}\nabla p - \nabla(gz) = 0$$

Taking the curl of this expression, and using the fact that $\nabla \cdot \mathbf{u}_g = 0$ and $\nabla \times \nabla f = 0$ then yields

$$(\mathbf{\Omega} \cdot \nabla)\mathbf{u} = 0$$

In other words, $\mathbf{u}$ does not vary in the direction $\mathbf{\Omega}$.

Taylor Columns

Taylor-Proudman Theorem:

If flow is geostrophic and homogeneous (ρ = constant) then the flow is two dimensional and does not vary in the direction of the rotation vector.

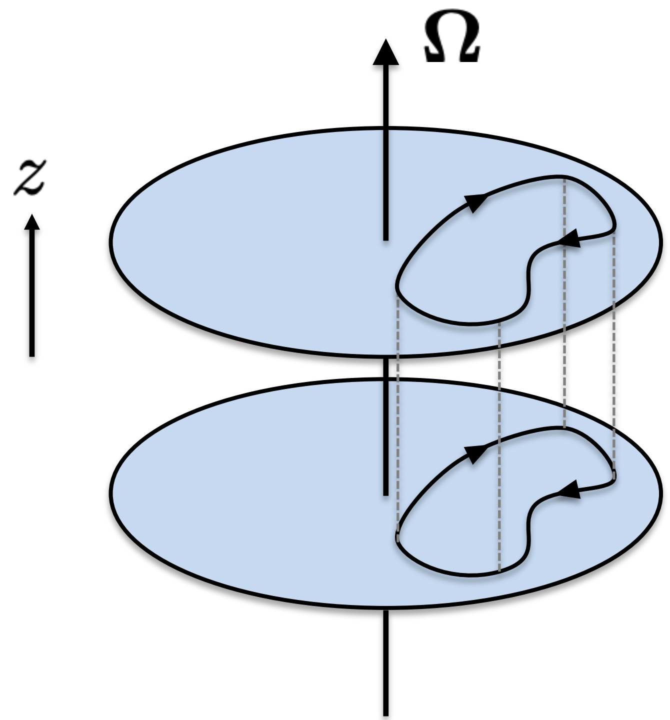


Figure: For slow, steady, frictionless flows the wind vectors do not vary along the direction of rotation.

Vertical columns of fluid (**Taylor columns**) remain vertical and undisturbed. These columns cannot be tilted.

Taylor Columns

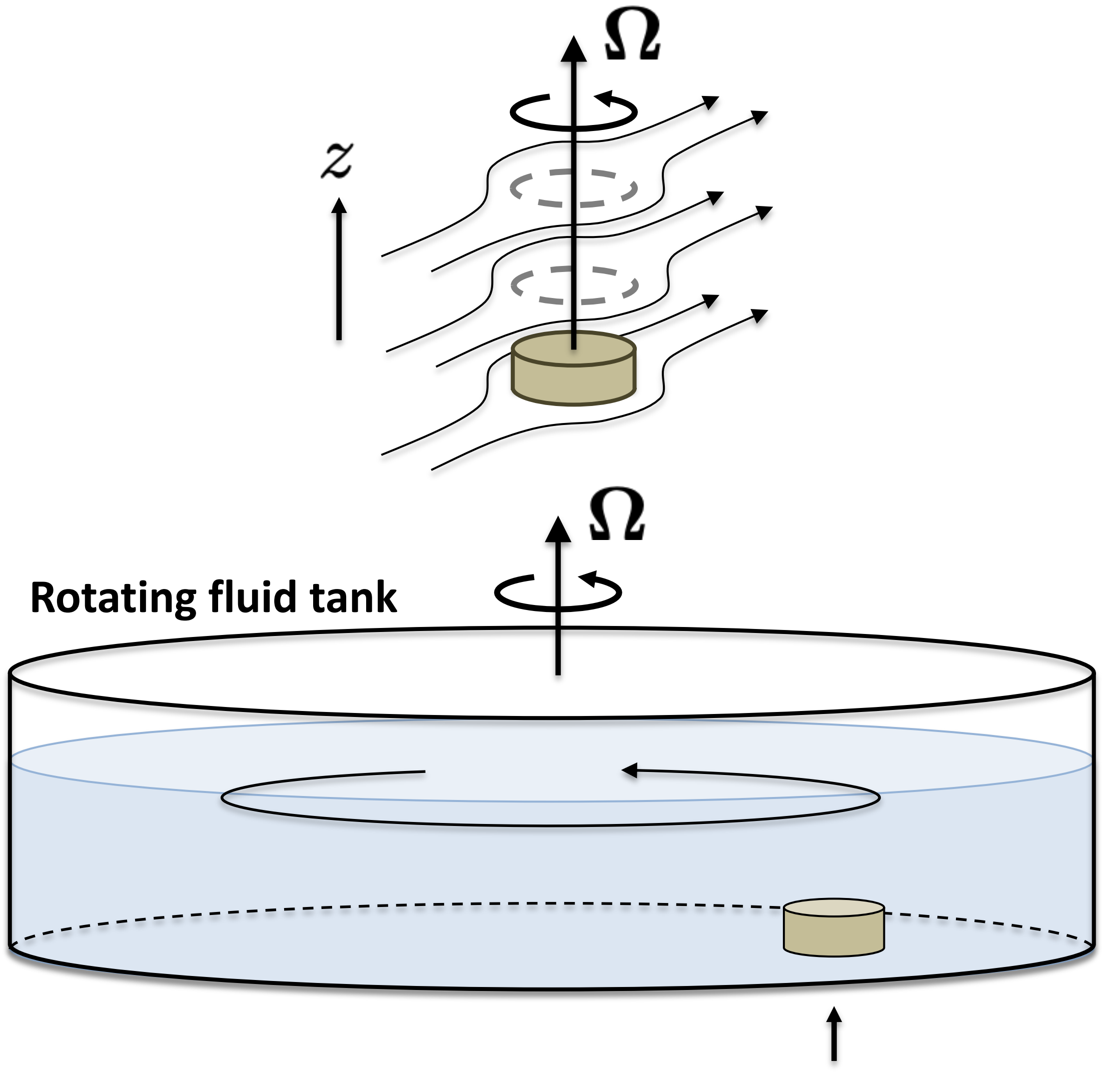


Figure: Under the Taylor-Proudman theorem flows must be along contours of constant fluid depth (otherwise the flow will lose its 2D character).

Placing an obstacle on the base of the tank in a rotating environment will reveal that the fluid above the obstacle will not mix with the environment.

Taylor Columns

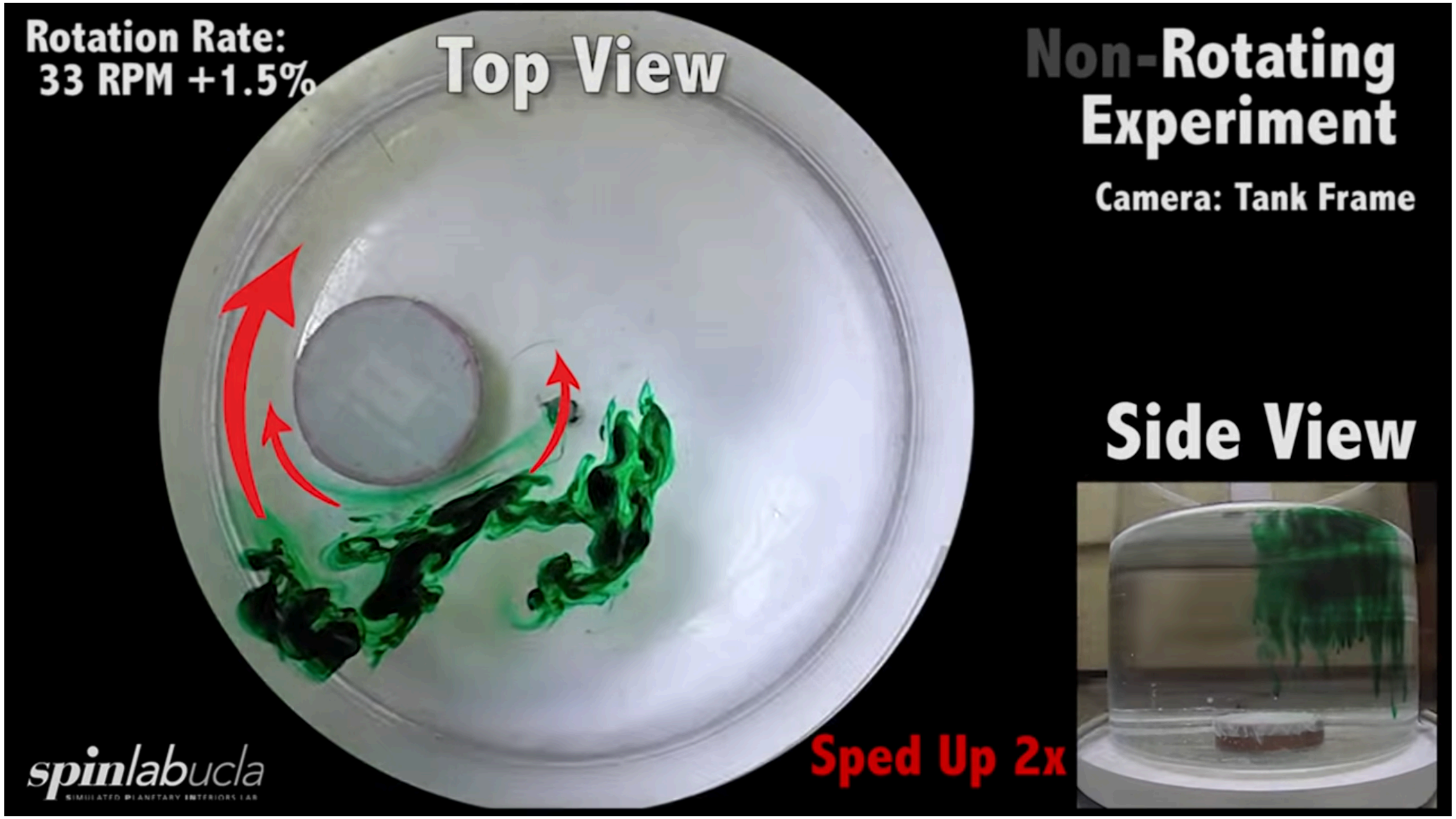


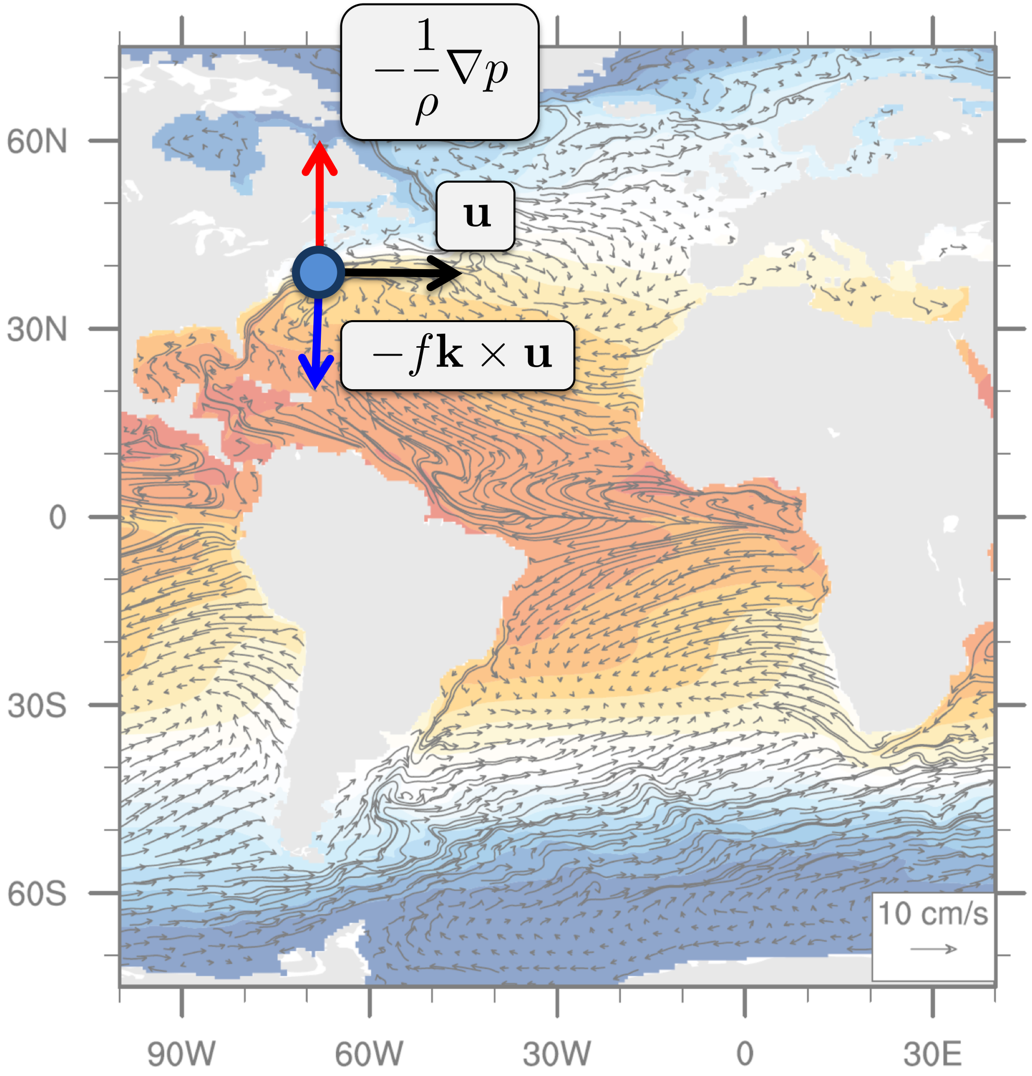
Figure: Taylor columns generated at the UCLA spinlab using a rotating turntable. https://www.youtube.com/watch?v=7GGfsW7gOLI

Near-Surface Flow and Flow at Depth

Near-Surface Flow

Figure: Consider a fluid parcel in the Gulf Stream being transported eastward.

To compensate for the southward-directed Coriolis force there must be a pressure gradient force directed northward.



Hydrostatic Balance

$$\frac{\partial p}{\partial z} = -g(\rho_{\text{ref}} + \sigma)$$

This equation can be integrated as:

$$\int_p^{p_s} dp = -g \int_z^{\eta} (\rho_{\text{ref}} + \sigma) dz$$

z depth

Where η is the height of the Ocean's free surface, where $p(\eta) = p_s$

$$p(z) = p_s + g\langle\rho\rangle(\eta - z)$$

Mean layer density

$$\langle\rho\rangle = \int_z^{\eta} (\rho_{\text{ref}} + \sigma) dz$$

Near-Surface Geostrophic Flow

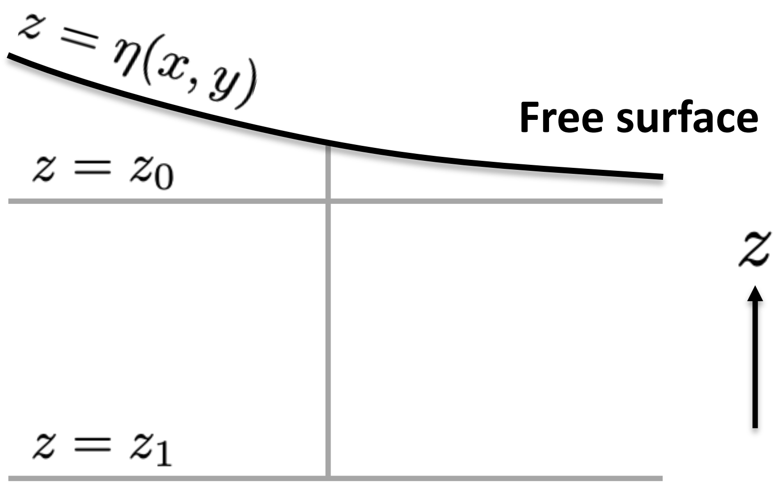


Figure: Schematic showing the height of the free surface of the ocean and two reference horizontal surfaces. One at the near surface ($z = z_0$) and one at depth ($z = z_1$).

From hydrostatic balance we have

$$p(z) = p_s + g\langle\rho\rangle(\eta - z)$$

In the **near-surface**, fractional variations in column depth are much greater than those of density (so we neglect the latter):

$$p(z) = p_s - g\rho_{\text{ref}}(z - \eta)$$

Horizontal variations in pressure depend on (1) variations in atmospheric pressure and (2) variations in free-surface height.

Near-Surface Geostrophic Flow

Ignoring gradients in atmospheric surface pressure, since these occur on much shorter timescales:

$$p(z) = p_s - g\rho_{\text{ref}}(z - \eta) \quad \Rightarrow \quad \nabla p \approx g\rho_{\text{ref}} \nabla \eta$$

And so, from geostrophic balance,

$$(\mathbf{u}_g)_{surf} = \frac{g}{f}\mathbf{k} \times \nabla \eta$$

$$(u_g)_{surf} = -\frac{g}{f}\frac{\partial \eta}{\partial y} \qquad (v_g)_{surf} = \frac{g}{f}\frac{\partial \eta}{\partial x}$$

Analogous to geostrophic wind on constant geopotential surfaces

Near-Surface Geostrophic Flow

$$(u_g)_{surf} = -\frac{g}{f}\frac{\partial \eta}{\partial y}$$

In the atmosphere, geostrophic winds of 15 m/s are associated with tilts of pressure surfaces by about 800 m over a distance of 5000 km (verify).

Because oceanic flow is weaker than atmospheric flow, we expect to see much gentler tilt of the free surface.

$$\Delta \eta = \frac{fLU}{g}$$

$$U = 0.1 \text{ m s}^{-1}$$
$$f = 10^{-4} \text{ s}^{-1}$$
$$L = 10^{6} \text{ m}$$

Answer:

Sea Surface Height

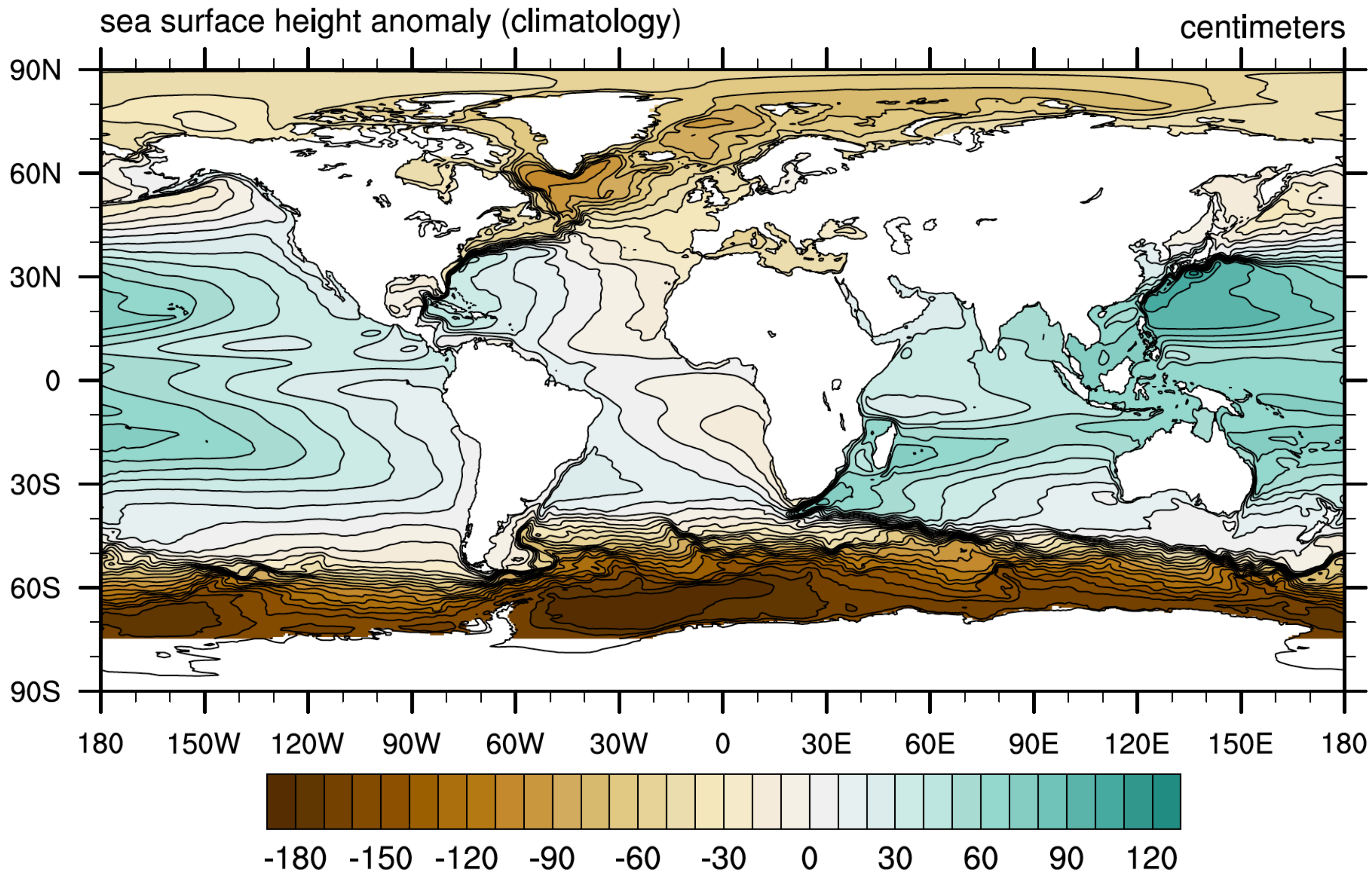


Figure: 10-year mean height of the sea surface relative to the geoid (contoured every 20 cm) as measured by satellite altimeter.

Geostrophic Flow at Depth

At depth (at $z = z_1$), variations in density can no longer be neglected compared to those of column depth.

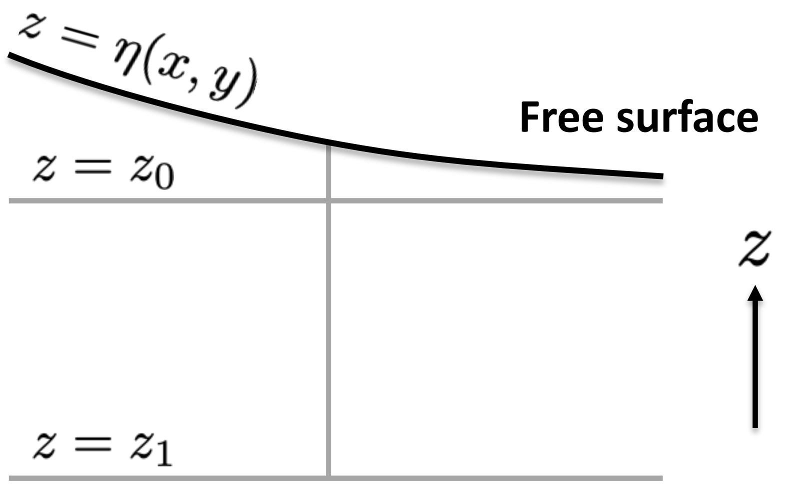
From earlier, for a given oceanic column:

$$p(z) = p_s + g\langle\rho\rangle(\eta - z)$$

Again ignoring variations in atmospheric surface pressure,

$$\nabla p \approx g\langle\rho\rangle\nabla\eta + g(\eta - z)\nabla\langle\rho\rangle$$

$$\Rightarrow \quad \mathbf{k} \times \nabla p = g\langle\rho\rangle\mathbf{k} \times \nabla\eta + g(\eta - z)\mathbf{k} \times \nabla\langle\rho\rangle$$



Geostrophic Flow at Depth

$$\mathbf{k} \times \nabla p = g\langle\rho\rangle\mathbf{k} \times \nabla\eta + g(\eta - z)\mathbf{k} \times \nabla\langle\rho\rangle$$

From the geostrophic wind relationship:

$$\mathbf{u}_g = \frac{1}{f\rho_{\text{ref}}}\mathbf{k} \times \nabla p$$

$$\mathbf{u}_g = \frac{1}{f\rho_{\text{ref}}}\left[\langle\rho\rangle\nabla\eta + g(\eta - z)\nabla\langle\rho\rangle\right]$$

Then using $\dfrac{\langle\rho\rangle}{\rho_{\text{ref}}} \approx 1$ gives

$$\mathbf{u}_g \approx \frac{g}{f}\mathbf{k} \times \nabla\eta + \frac{g(\eta - z)}{f\rho_{\text{ref}}}\mathbf{k} \times \nabla\langle\rho\rangle$$

Geostrophic flow in the interior ocean

Geostrophic Flow at Depth

$$\mathbf{u}_g \approx \frac{g}{f}\mathbf{k} \times \nabla \eta + \frac{g(\eta - z)}{f \rho_{\text{ref}}}\mathbf{k} \times \nabla \langle \rho \rangle$$

Geostrophic flow in the ocean

Observe that if **density is uniform** that the geostrophic velocity is purely a function of the free surface height and **independent of depth**.

In practice, geostrophic velocities are smaller at depth than at the surface, suggesting that these two terms balance each other.

Geostrophic Flow at Depth

$$\mathbf{u}_g \approx \frac{g}{f}\mathbf{k} \times \nabla \eta + \frac{g(\eta - z)}{f \rho_{\text{ref}}} \mathbf{k} \times \nabla \langle \rho \rangle$$

If geostrophic flow is nearly zero (as observed in the deep ocean), we must have that these two terms roughly balance each other:

$$\frac{H}{\rho_{\text{ref}}} |\nabla \langle \rho \rangle| \approx |\nabla \eta|$$

where H is the column height

Divide through by $\mathcal{N}^2 = -\frac{g}{\rho_E} \frac{\partial \sigma}{\partial z}$

$$|\nabla_\rho z| \approx \frac{g}{\mathcal{N}^2 H} \times |\nabla \eta|$$

Isopycnal (surfaces of constant density) slope

Free surface slope

Geostrophic Flow at Depth

$$|\nabla_\rho z| \approx \frac{g}{\mathcal{N}^2 H} \times |\nabla \eta|$$

Using

$$\left.\begin{aligned} \mathcal{N} &\approx 5 \times 10^{-3}\ \mathrm{s}^{-1} \\ H &= 1\ \mathrm{km} \end{aligned}\right\} \quad \frac{g}{\mathcal{N}^2 H} \approx 400$$

➡ For every meter the free surface tilts up, density surfaces must tilt down by about 400m.

Geostrophic Flow at Depth

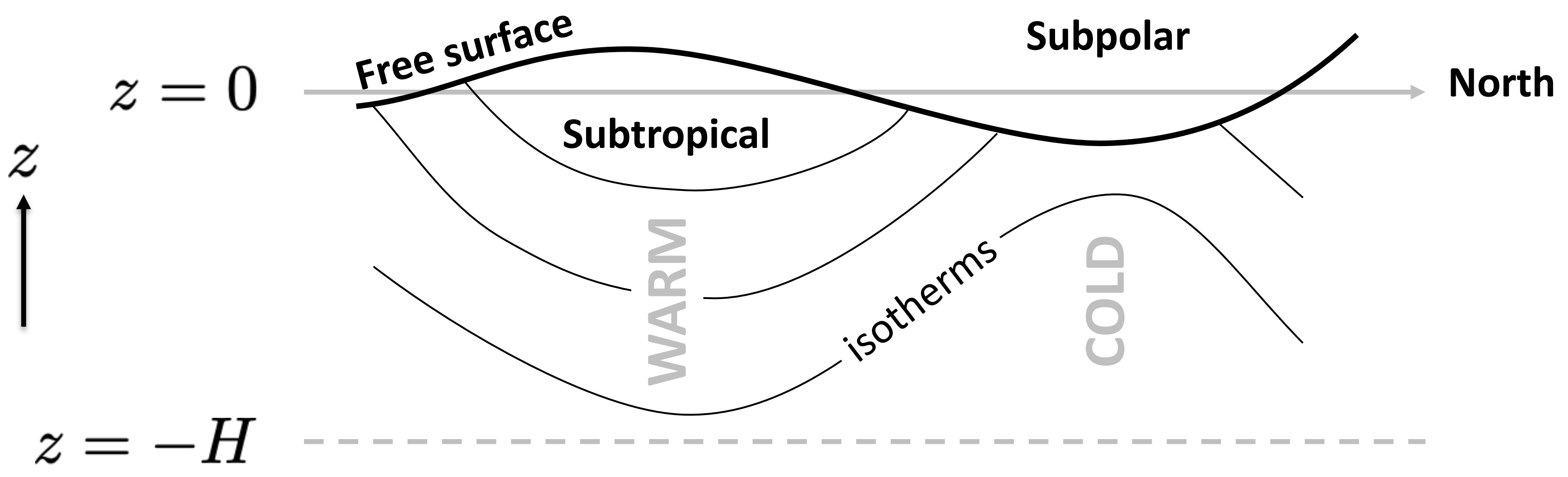


Figure: Warm subtropical columns of fluid expand relative to colder polar columns. Thus the sea surface (measured relative to the geoid) is higher, by about 1 m, in the subtropics than at the poles.

Pressure gradients associated with sea surface tilt are largely compensated almost exactly by vertical thermocline undulations. Consequently the abyssal circulation is much weaker than the surface.

Sea Surface Height

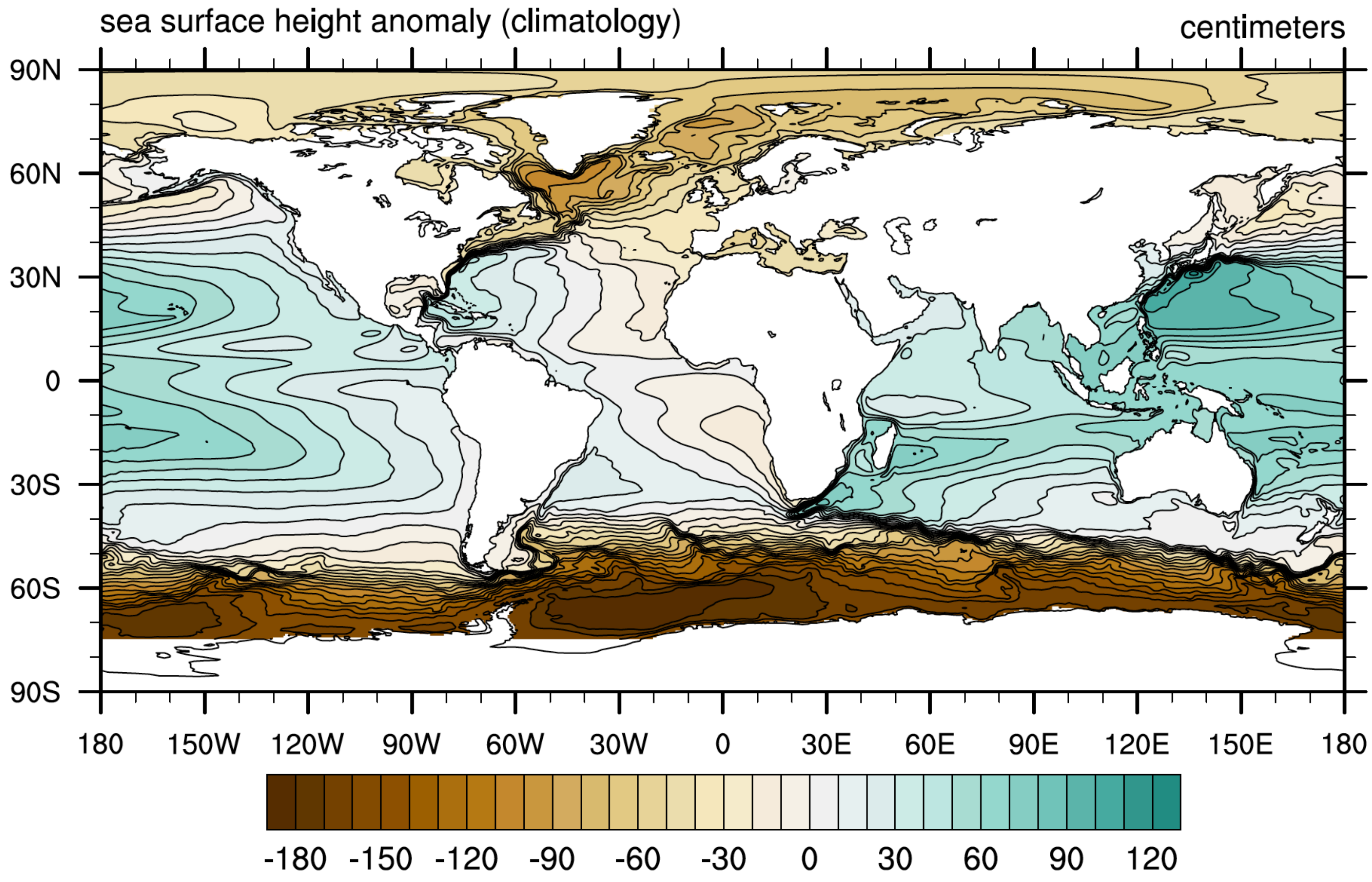
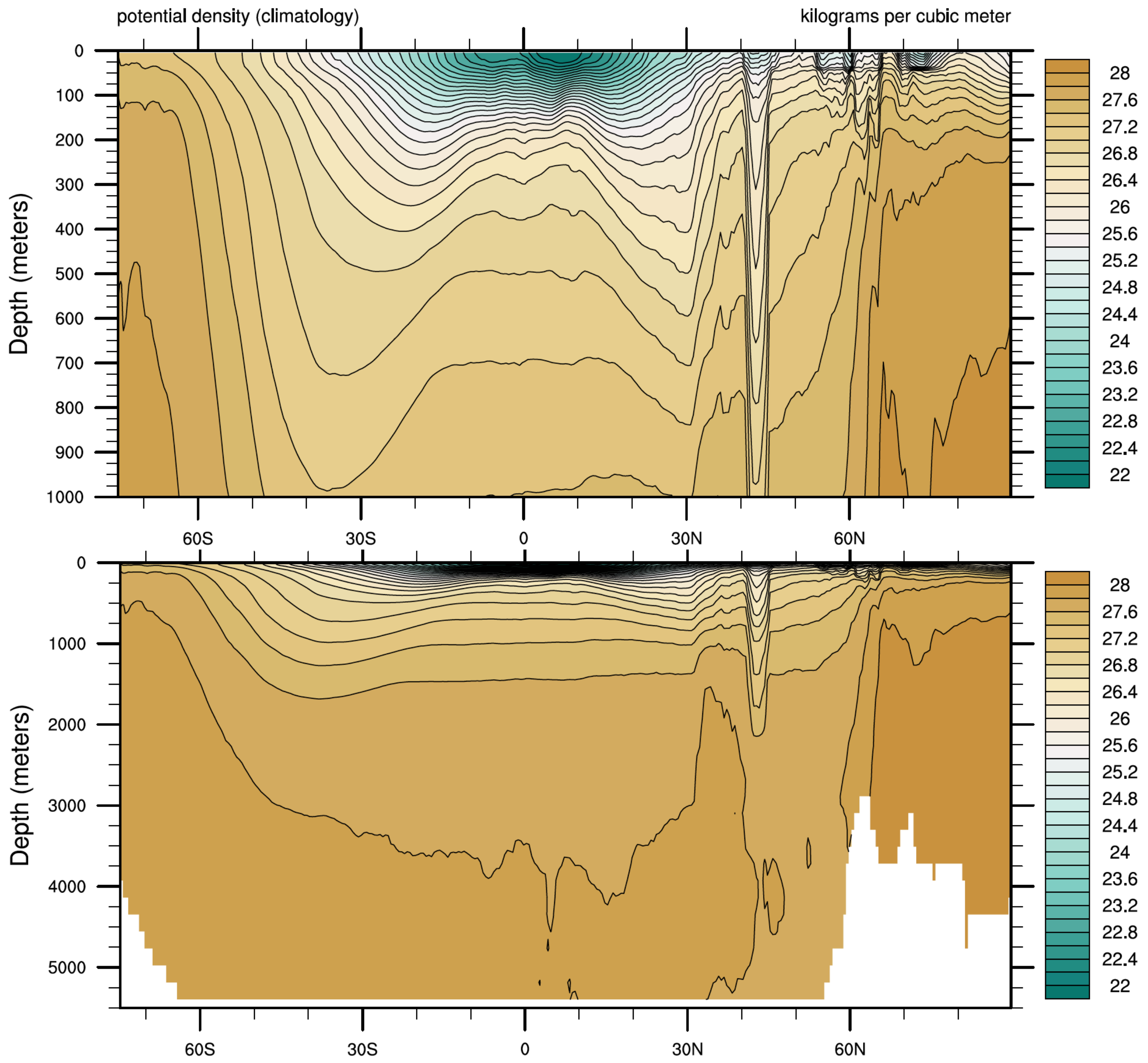


Figure: 10-year mean height of the sea surface relative to the geoid (contoured every 20 cm) as measured by satellite altimeter.

Potential Density

Figure: Zonal average annual-mean potential density anomaly in the world oceans. Note that darker colors indicate less dense fluid. Compare with zonal average annual-mean temperature.

$$\sigma = \rho - \rho_{ref}$$



Depth of the 1026.5 kg/m3 Density Surface

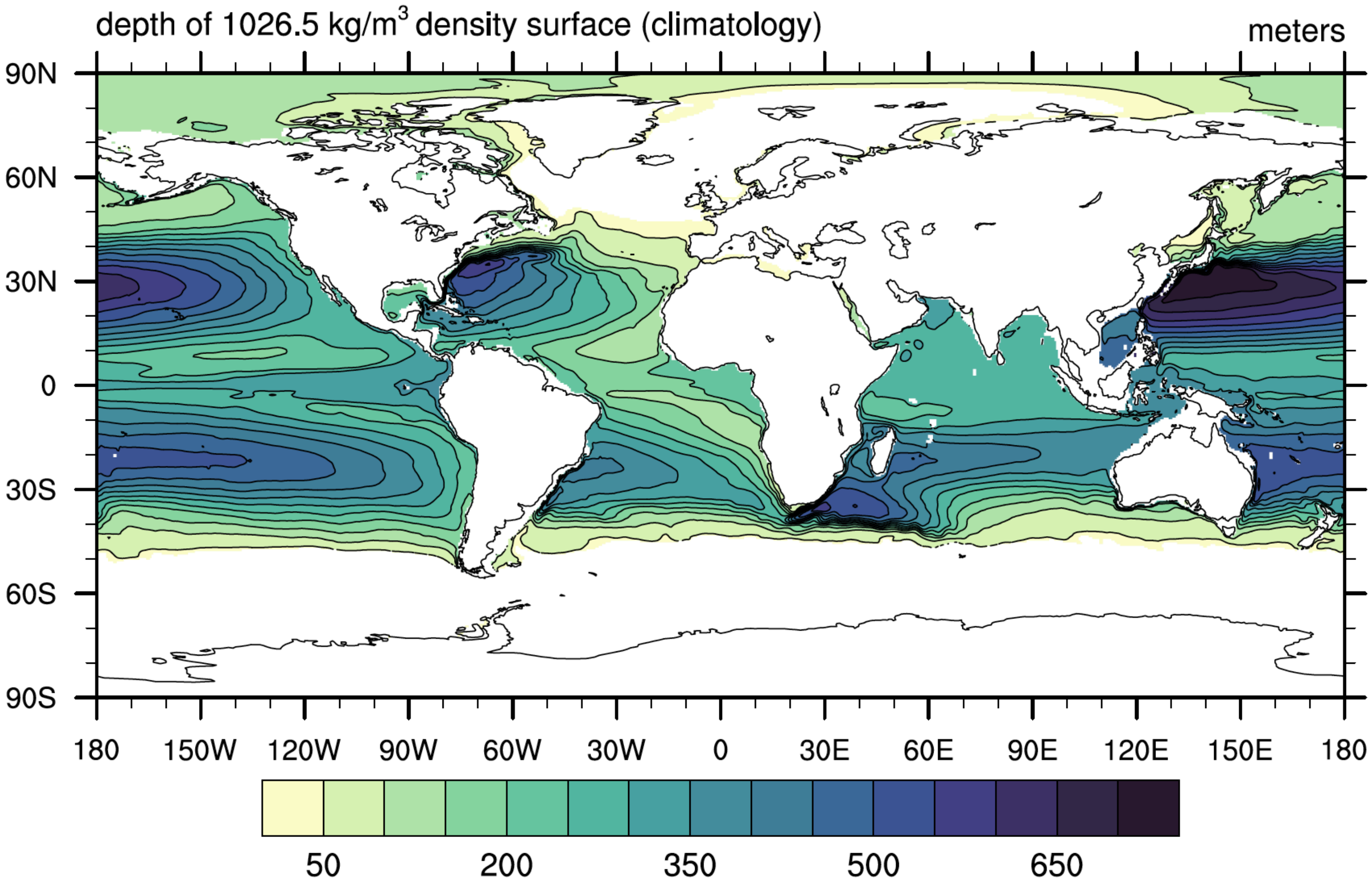


Figure: Depth in meters of the annual mean σ = 26.5 kg m^{-3} surface over the global ocean.

Steric Effects

The spatial variations in the height of the ocean surface are controlled to a large degree by expansion of warm ocean water and contraction of cold ocean water (observe that sea surface heights are high over the tropics and low over the polar regions).

Further, salty columns of water are shorter than fresh columns (temperature and pressure being equal).

Definition: Expansion and contraction of water columns due to T and S anomalies is known as the **steric effect**.

Steric Effects

Using $$\frac{H}{\rho_{\text{ref}}} |\nabla \langle \rho \rangle| \approx |\nabla \eta|$$

And averaged equation of state

$$\sigma(T,S) \approx \sigma_0 + \rho_{\text{ref}} \Big(-\alpha_T [T - T_0] + \beta_S [S - S_0] \Big)$$

$$\Rightarrow \quad \langle \rho \rangle \approx \rho_{\text{ref}} [1 - \alpha_T \langle T - T_0 \rangle + \beta_S \langle S - S_0 \rangle]$$

$$\Rightarrow \quad \nabla \langle \rho \rangle \approx \rho_{\text{ref}} [-\alpha_T \nabla \langle T \rangle + \beta_S \nabla \langle S \rangle]$$

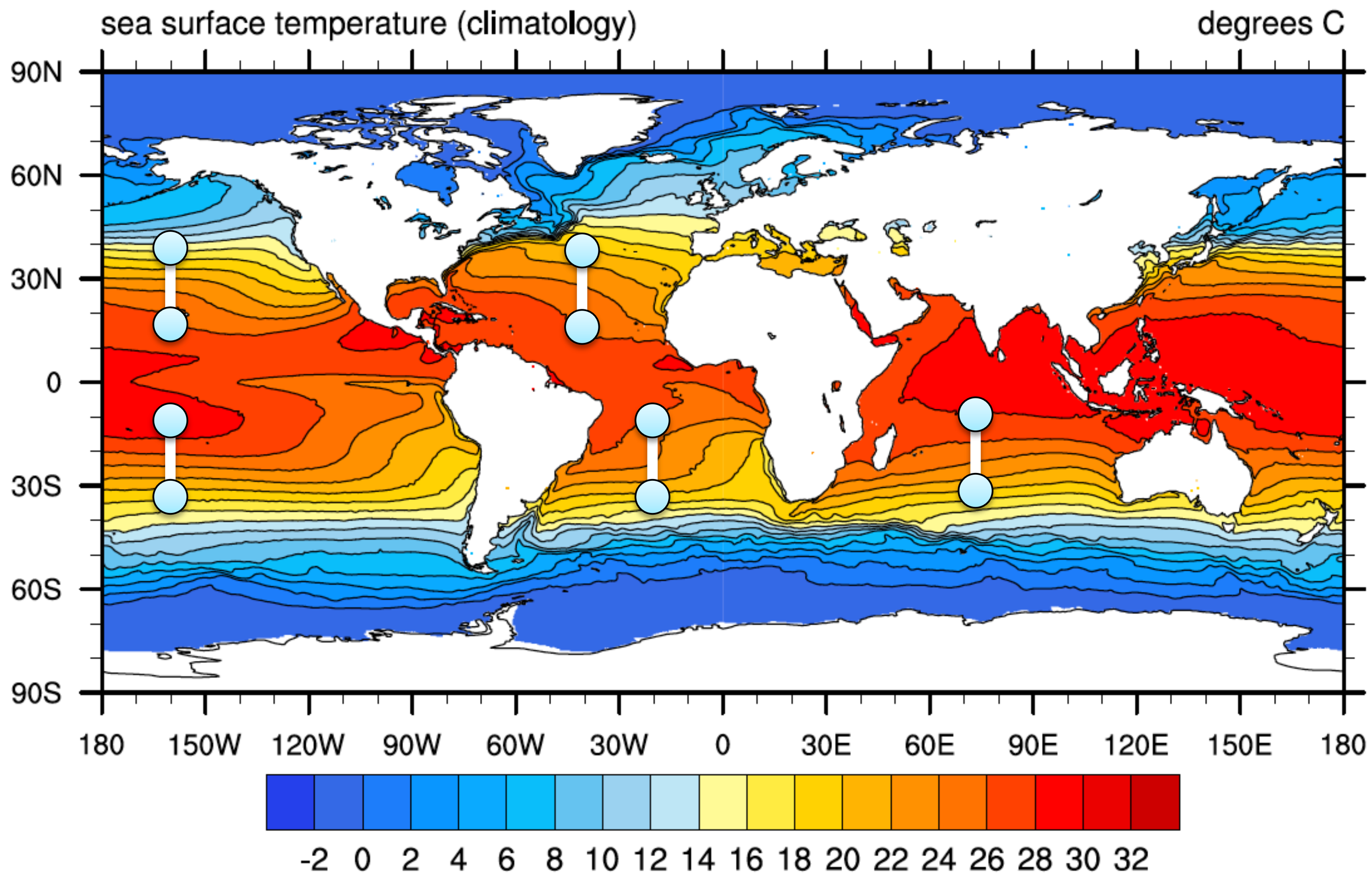
$$\Rightarrow \quad \frac{\Delta \eta}{H} \approx \alpha_T \Delta T - \beta_S \Delta S$$

Taking into account the fact that temperature increases thickness and salinity decreases thickness.

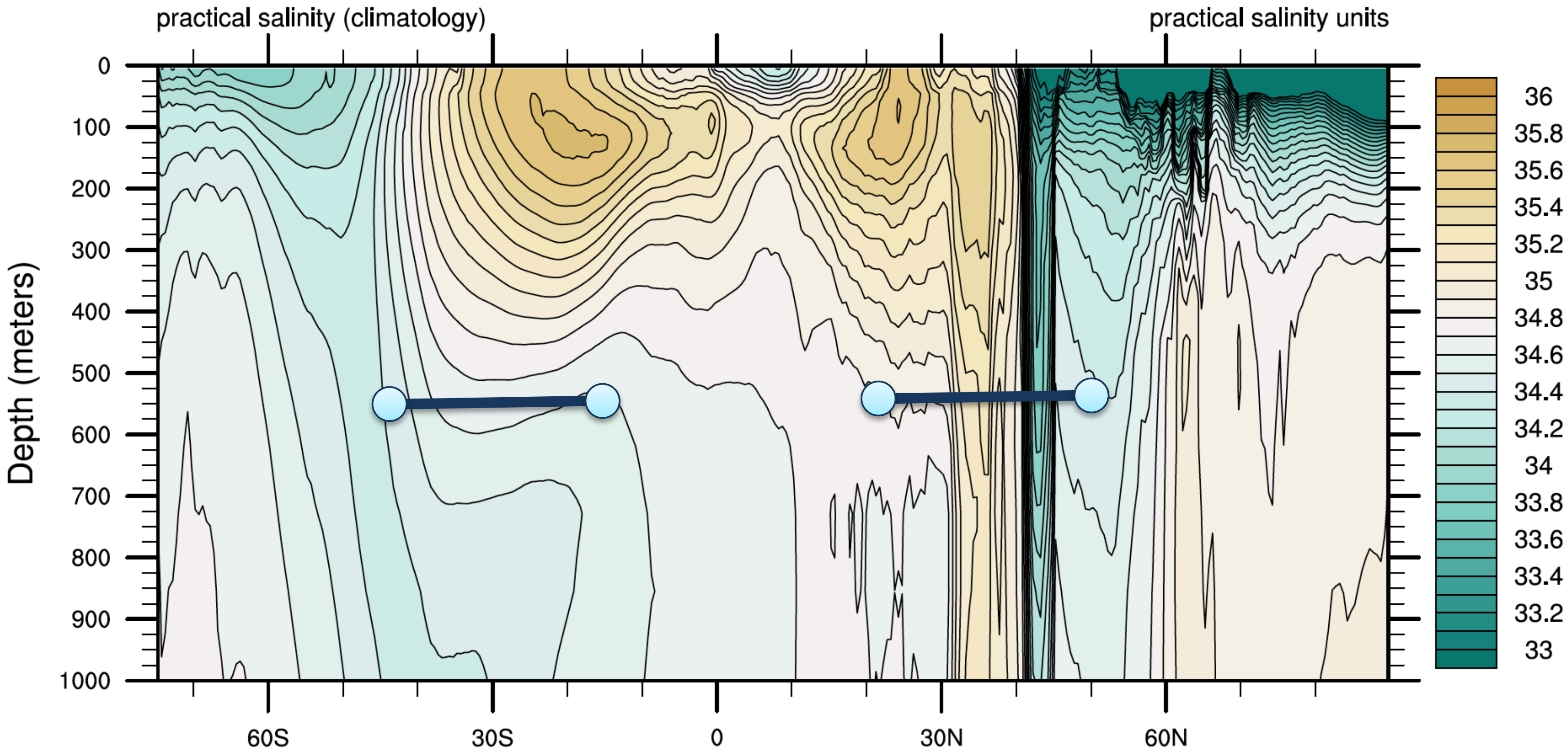
Steric Effects

Over the meridional extent of the subtropical ocean gyres, we estimate

$$\Delta T \approx 10^\circ C$$



Steric Effects



Over the meridional extent of the subtropical ocean gyres, we estimate

$$\Delta S \approx 0.5 \text{ psu}$$

Steric Effects

$$\frac{\Delta\eta}{H} \approx \alpha_T \Delta T - \beta_S \Delta S$$

$$\Delta T \approx 10^\circ C$$

$$\Delta S \approx 0.5 \text{ psu}$$

$$\alpha_T \approx 2.0 \times 10^{-4} \ {}^\circ\text{C}^{-1}$$

$$\beta_S \approx 7.6 \times 10^{-4} \ \text{psu}^{-1}$$

$$\frac{\Delta\eta}{H} \approx \left(\ 2 \quad + \quad (-0.38) \ \right) \times 10^{-3}$$

Temperature expansion (2)

Salinity contraction (−0.38)

So over the top kilometer of the column, height variations due to salt are more than offset by approximately 2m of thermal expansion.

Sea Surface Height

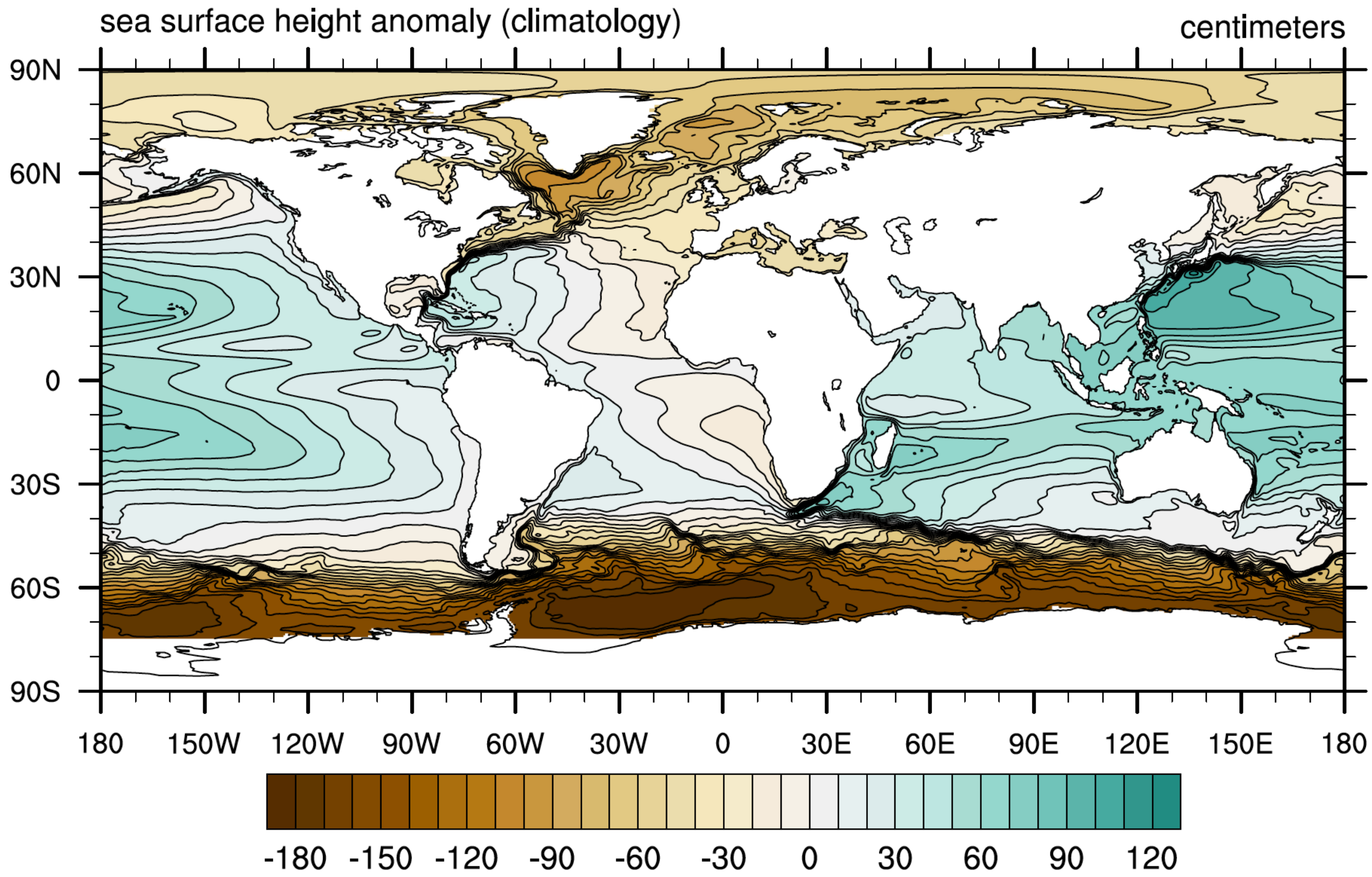


Figure: SODA3 report mean height of the sea surface relative to the geoid (contoured every 10 cm).

ATM 241 Climate Dynamics
Lecture 8
Ocean Dynamics

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Thank You!