ATM 241, Spring 2020
Lecture 4b
Dry Convection

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Marshall & Plumb
Ch. 4

In this section…

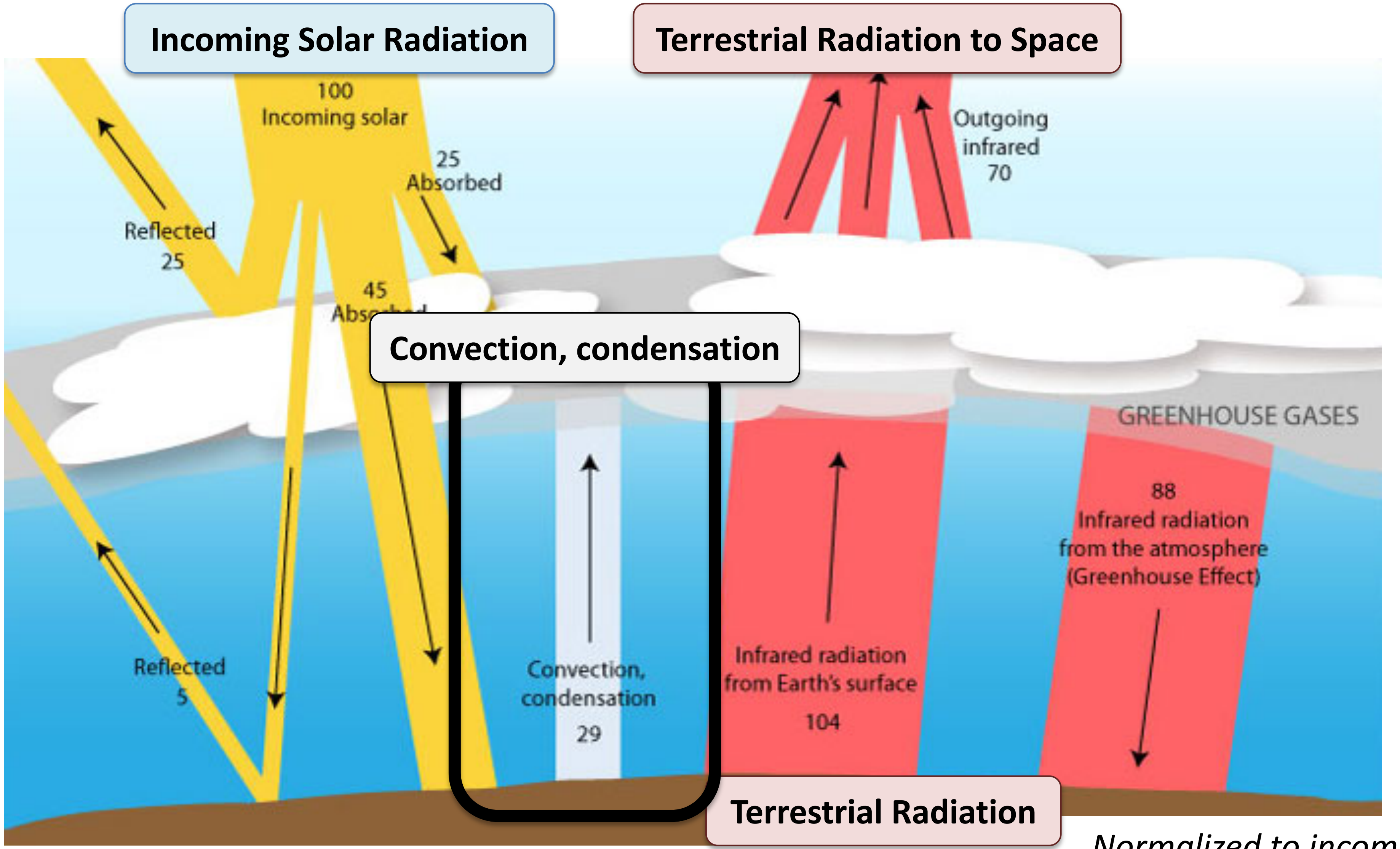
Definitions

- Rayleigh-Bénard convection
- Unstable atmospheric conditions
- Stable atmospheric conditions
- Brunt-Väisälä Frequency
- Mountain lee waves
- Temperature inversion

Questions

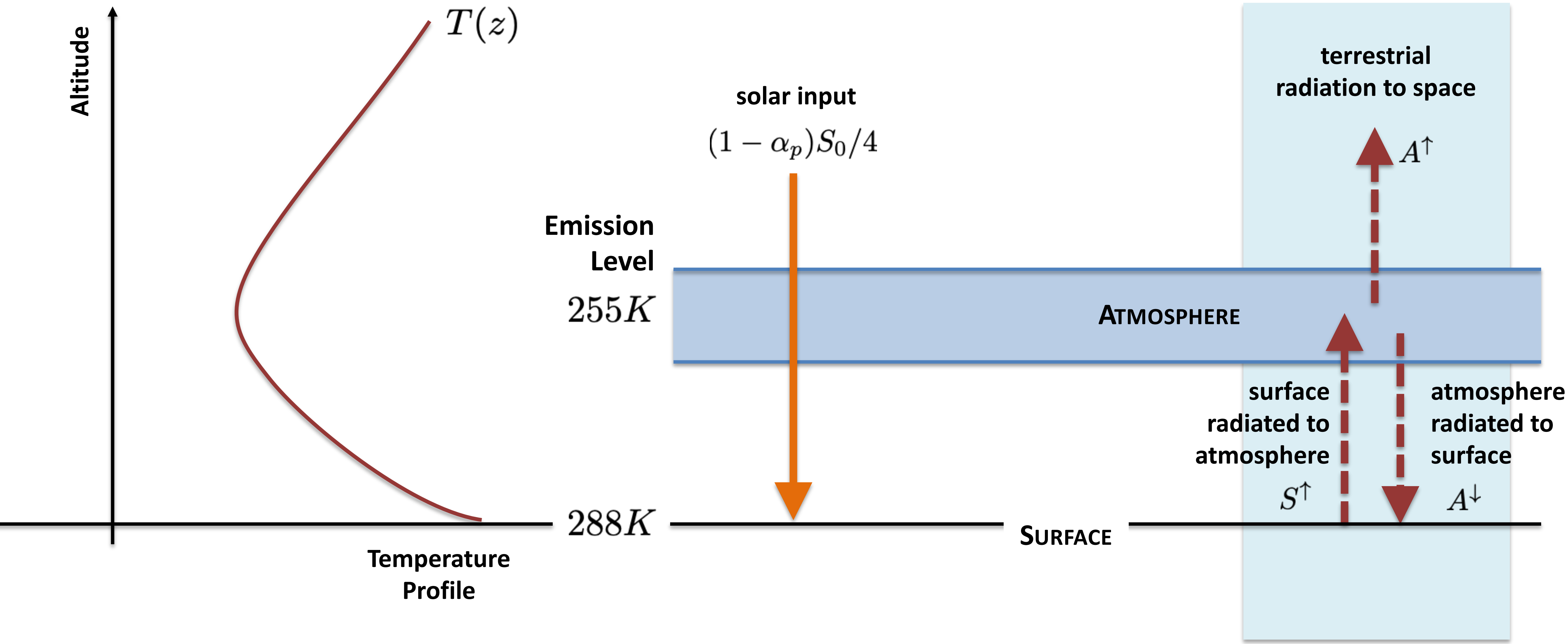
- What role does convection play in the energy budget?
- When / why does convection occur?
- What are the characteristics of convection?
- In the context of the parcel model, what is the meaning of stability / instability?
- What are the conditions for environmental stability and instability?
- What atmospheric features arise under stable and unstable conditions?

Energy Balance

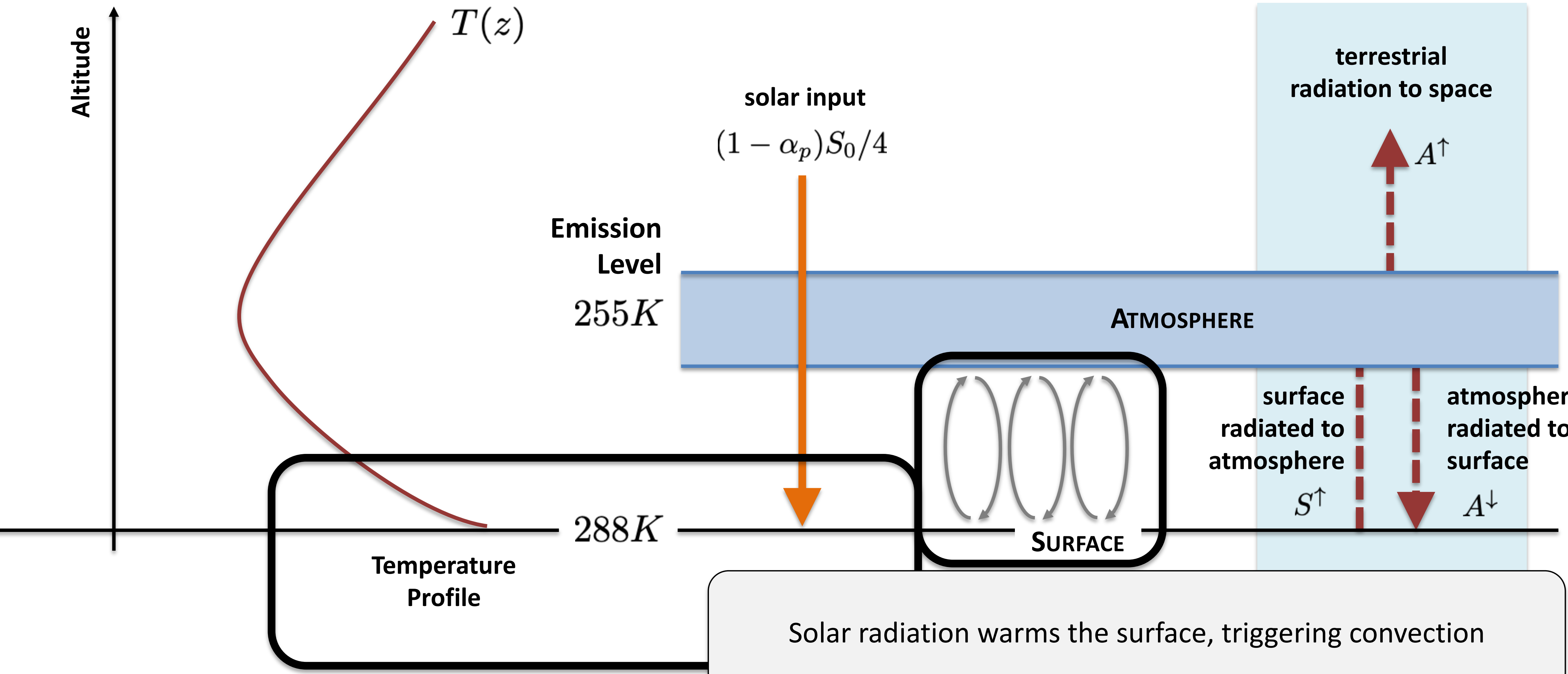


Normalized to incoming solar = 100

Convection

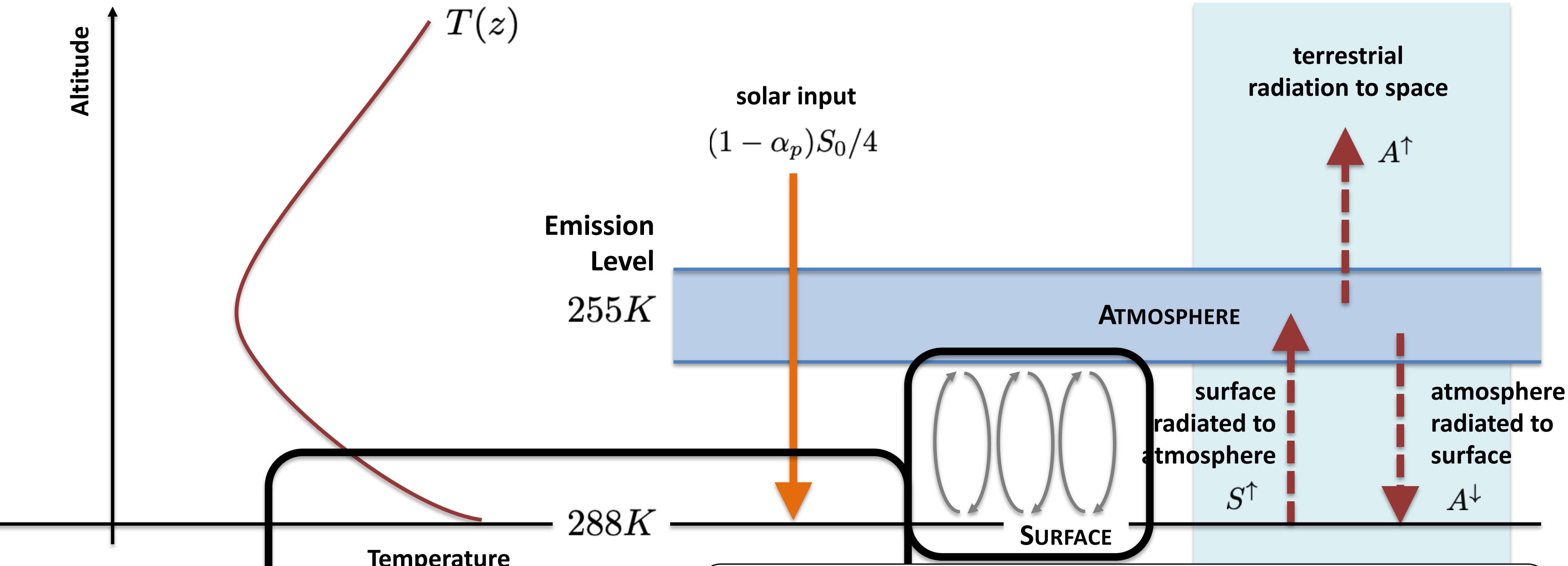

Altitude
$T(z)$
Temperature Profile
solar input
$(1-\alpha_p)S_0/4$
Emission Level
$255K$
$288K$
terrestrial radiation to space
$A^{\uparrow}$
ATMOSPHERE
surface radiated to atmosphere
$S^{\uparrow}$
atmosphere radiated to surface
$A^{\downarrow}$
SURFACE

Convection



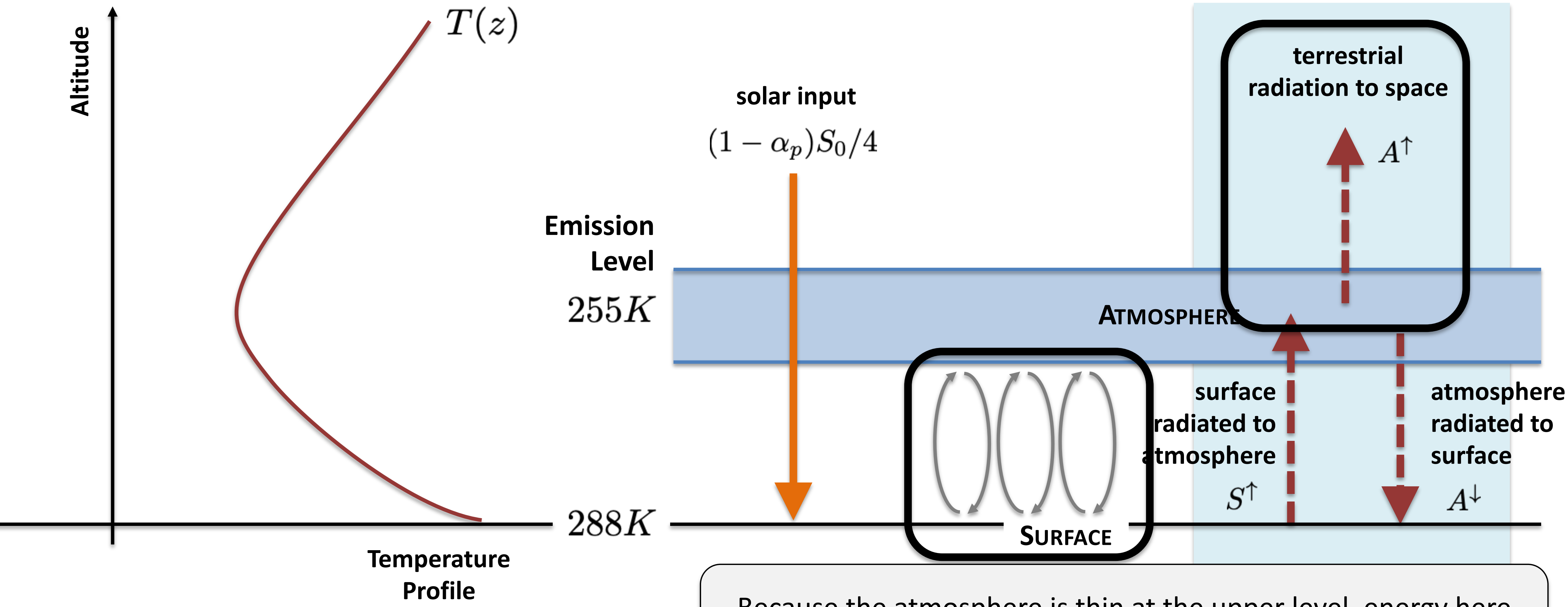
Solar radiation warms the surface, triggering convection

Convection



Convection exchanges the high potential temperature air at the surface with low potential temperature air aloft.

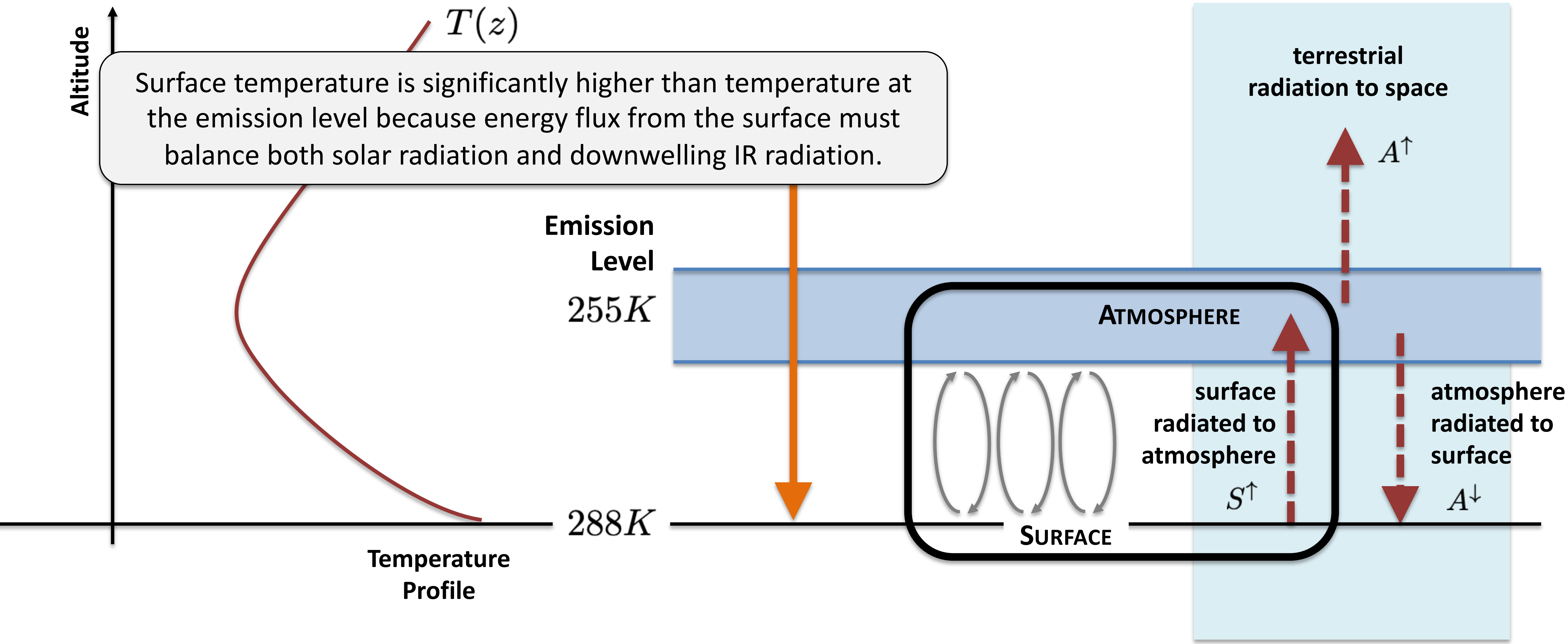
Convection



Because the atmosphere is thin at the upper level, energy here can be radiated out to space.

Convection

Surface temperature is significantly higher than temperature at the emission level because energy flux from the surface must balance both solar radiation and downwelling IR radiation.



Convection

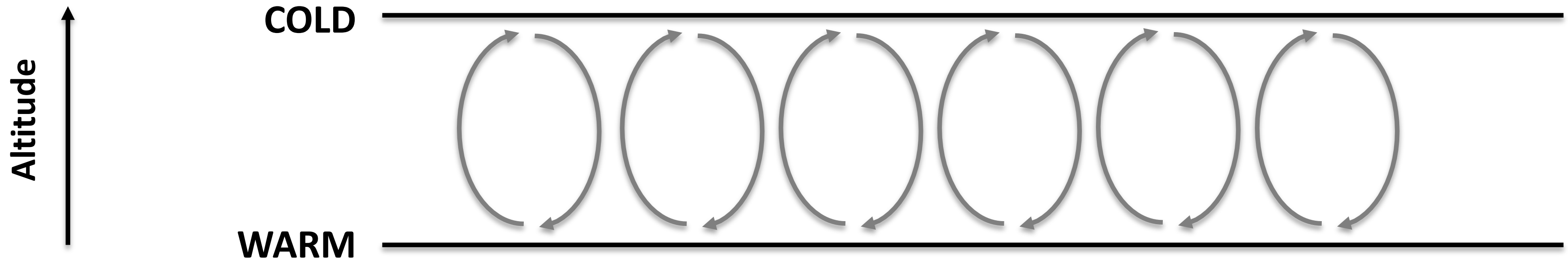


Figure: A schematic of shallow convection in a fluid, such as air, triggered by warming from below and/or cooling from above. The convective pattern that develops in this situation is known as **Rayleigh-Bénard Convection**.

A fluid heated in such a manner tends to develop overturning motions since a decrease in temperature corresponds to an increase in density, leading to the fluid being top-heavy.

Ideal gas law

$$\rho = \frac{p}{RT}$$

https://en.wikipedia.org/wiki/Rayleigh%E2%80%93B%C3%A9nard_convection

Convection

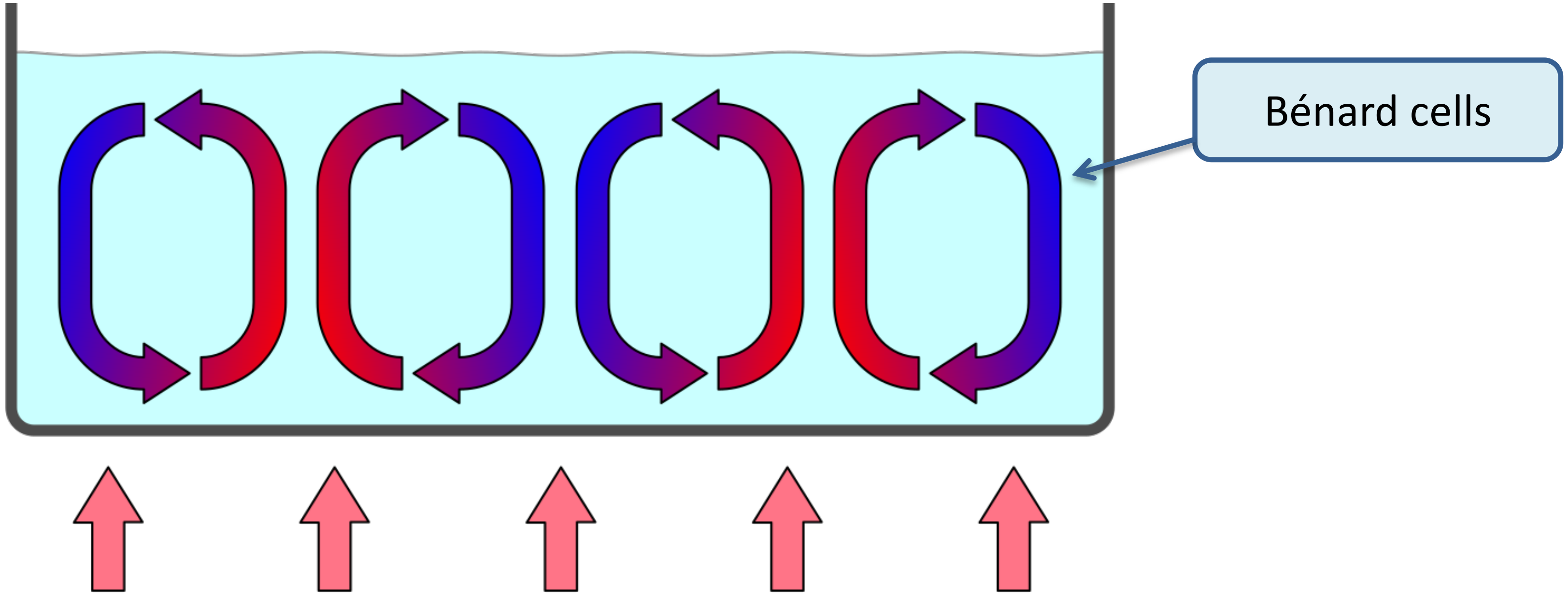


Figure: Schematic of shallow convection in a fluid triggered by warming below.

https://en.wikipedia.org/wiki/Convection

Convection

Two questions:

- Why do motions develop when the equilibrium state just discussed has no net forces anywhere?
- Why are the motions horizontally inhomogeneous when the external forcing (the heating) is uniform?

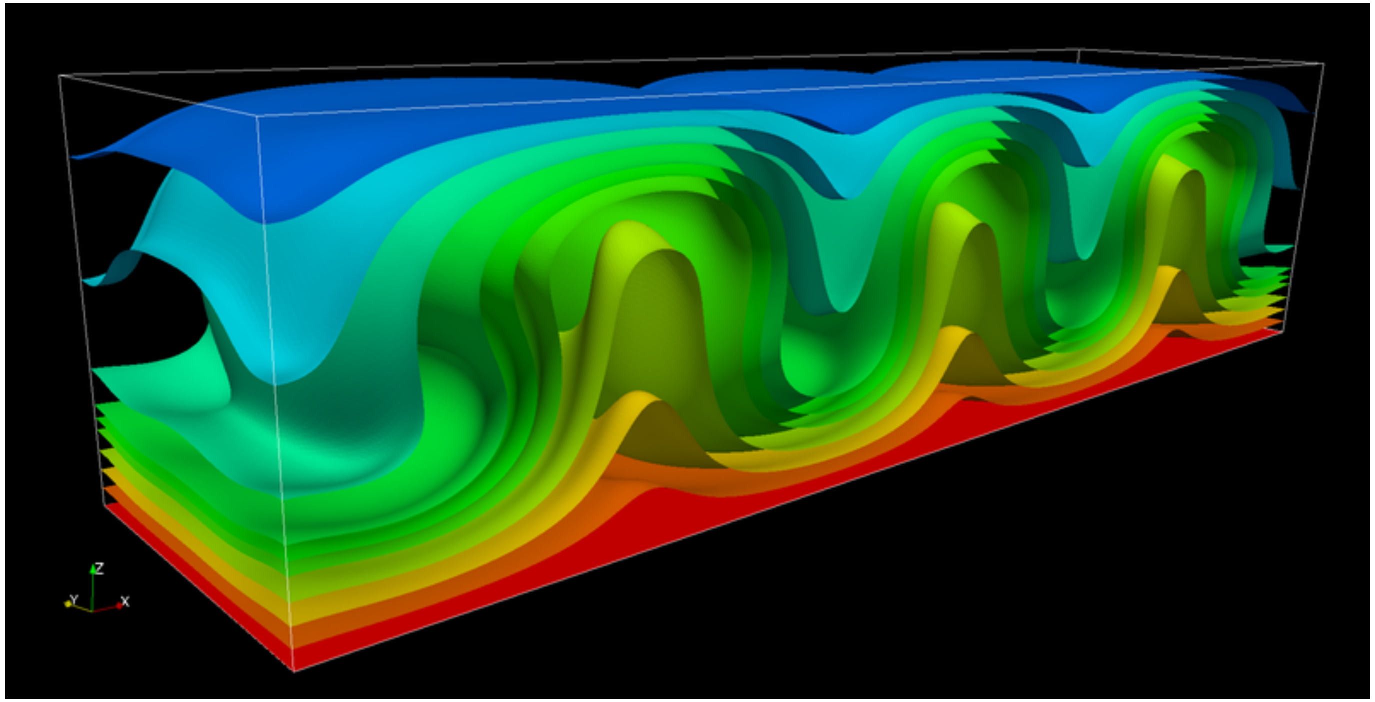


Figure: 3D simulation of Rayleigh-Bénard convection.

https://commons.wikimedia.org/wiki/File:Ray_40MTet.png

Rising Warm Bubble

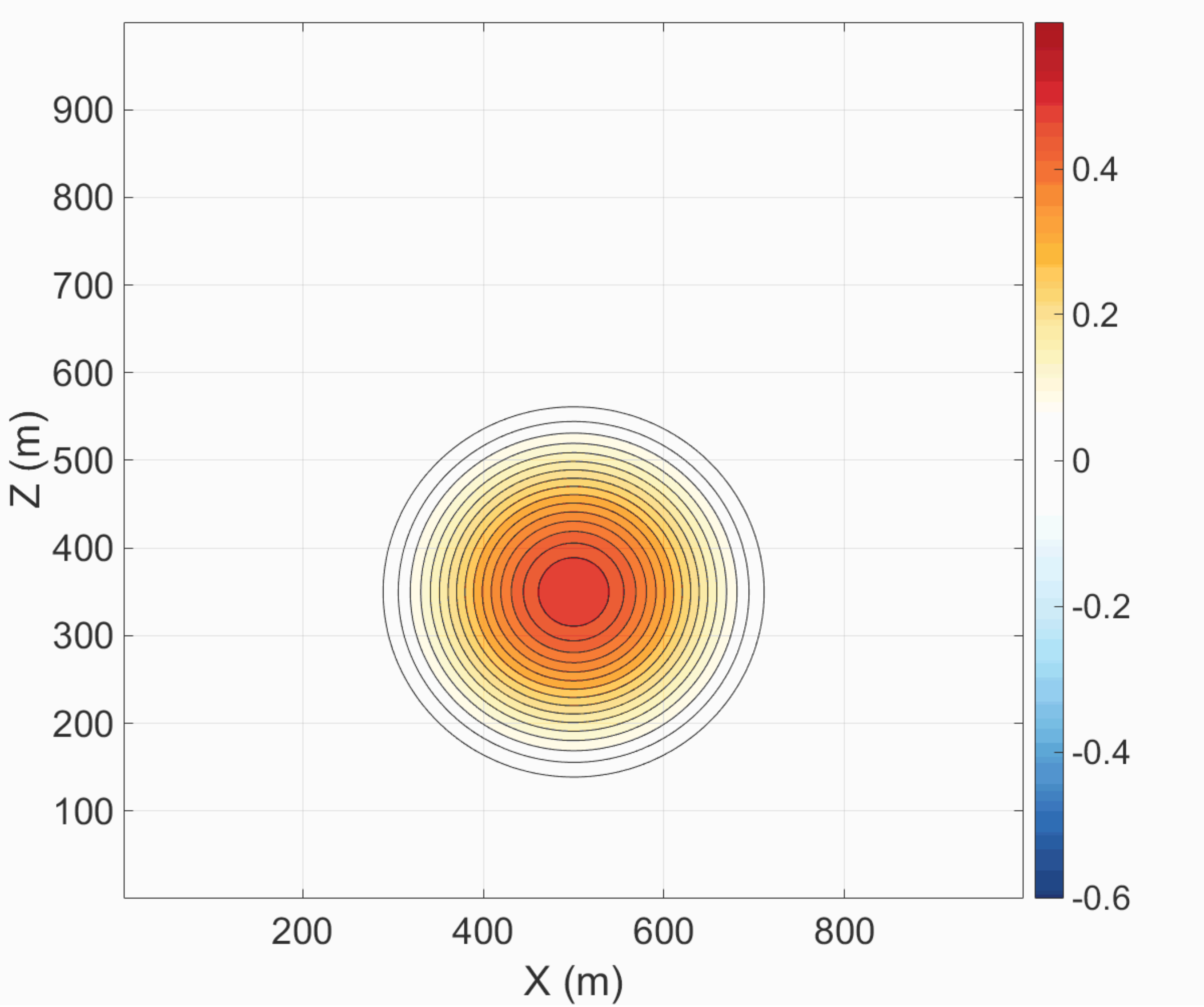


Figure: The temperature perturbation for an idealized "warm bubble" experiment with rigid lid at 1000m altitude.

Convection

Two questions:

- Why do motions develop when the equilibrium state just discussed has no net forces anywhere?
- Why are the motions horizontally inhomogeneous when the external forcing (the heating) is uniform?

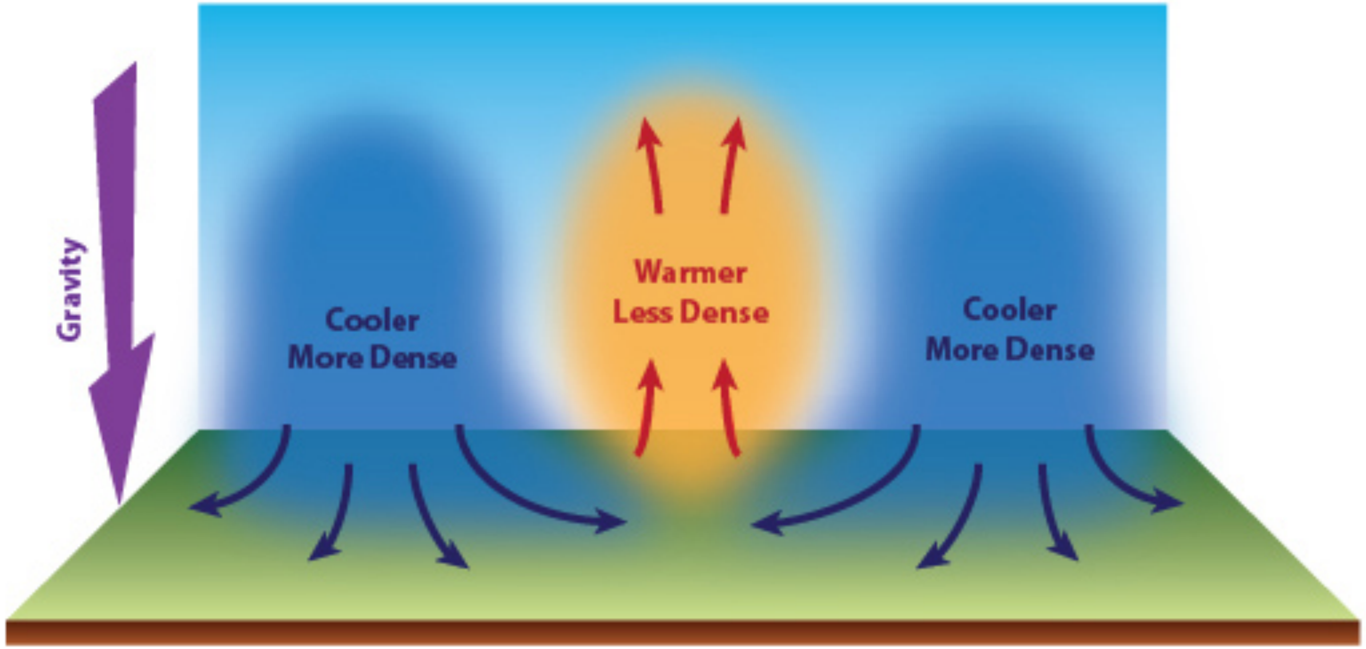


Figure: A depiction of air exchange that occurs in and around a convective plume.

https://www.weather.gov/jetstream/parcels

The Nature of Instability

Question: Why do motions develop when the equilibrium state just discussed has no net forces anywhere?

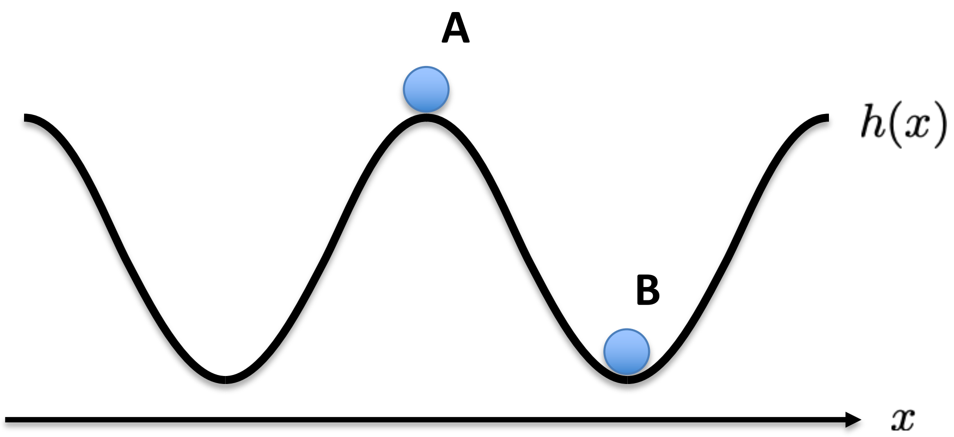
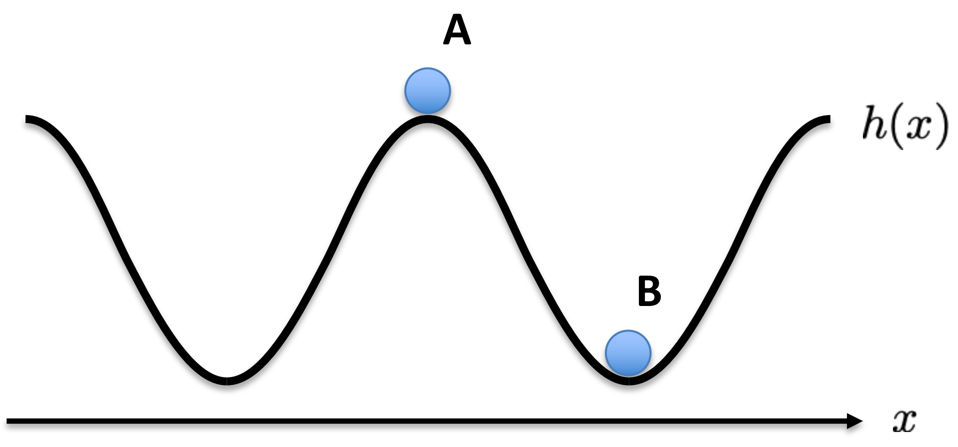


Figure: Although balls resting states A and B are at "equilibrium" small perturbations will have dramatically different outcomes.

The Nature of Instability



In this scenario, the ball will experience an acceleration proportional to the slope:

$$a = -g \frac{dh}{dx}$$

Consider "small perturbations" about an equilibrium state, expanded via Taylor series:

$$\frac{dh}{dx}(x_{eq} + \delta x) \approx \frac{dh}{dx}(x_{eq}) + \left(\frac{d^2 h}{dx^2} \right)_{eq} \delta x$$

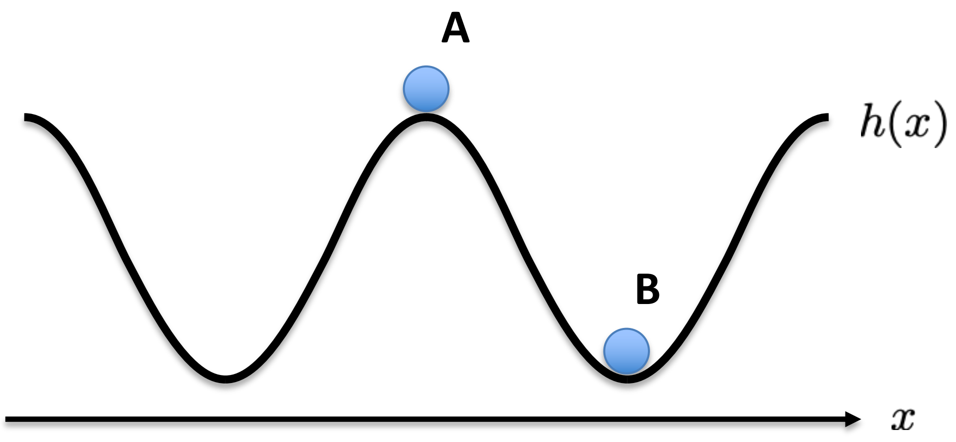
But at an equilibrium we have $\frac{dh}{dx}(x_{eq}) = 0$

And so $$\frac{dh}{dx}(x_{eq} + \delta x) \approx \left(\frac{d^2 h}{dx^2} \right)_{eq} \delta x$$

The Nature of Instability

$$a = -g \frac{dh}{dx}$$

$$\frac{dh}{dx}(x_{eq} + \delta x) \approx \left(\frac{d^2 h}{dx^2} \right)_{eq} \delta x$$



Velocity is equal to the time derivative of the offset:

$$u = \frac{d(\delta x)}{dt}$$

Accleration is equal to the time derivative of the velocity:

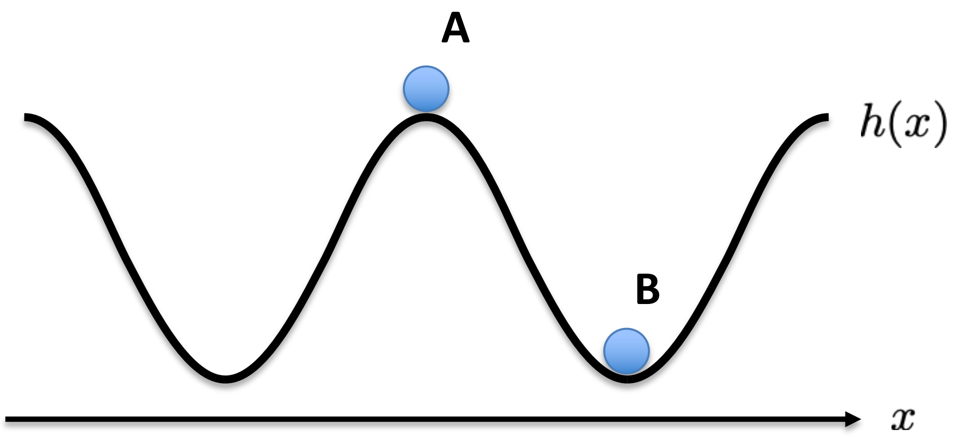
$$a = \frac{d^2(\delta x)}{dt^2}$$

$$\frac{d^2(\delta x)}{dt^2} = -g \left(\frac{d^2 h}{dx^2} \right)_{eq} \delta x$$

Does this look familiar?

The Nature of Instability

$$\frac{d^2(\delta x)}{dt^2} = -g\left(\frac{d^2 h}{dx^2}\right)_{eq} \delta x$$



Note the differences between equilibrium points A and B:

$$\left(\frac{d^2 h}{dx^2}\right)_A < 0 \quad \Longrightarrow \quad -g\left(\frac{d^2 h}{dx^2}\right)_A > 0$$

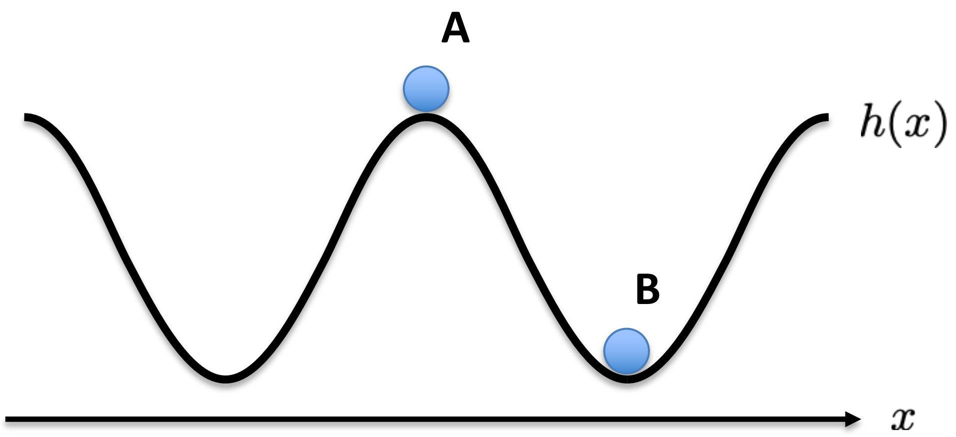
A positive value of δx will lead to a positive acceleration (a speedup)

$$\left(\frac{d^2 h}{dx^2}\right)_B > 0 \quad \Longrightarrow \quad -g\left(\frac{d^2 h}{dx^2}\right)_B < 0$$

A positive value of δx will lead to a negative acceleration (a slowdown).

The Nature of Instability

$$\frac{d^2(\delta x)}{dt^2} = -g \left(\frac{d^2 h}{dx^2} \right)_{eq} \delta x$$



Unstable Solution:

$$\left(\frac{d^2 h}{dx^2} \right)_{eq} < 0 \qquad \delta x = c_1 e^{\sigma_+ t} + c_2 e^{\sigma_- t} \qquad \sigma_\pm = \pm \sqrt{-g \left(\frac{d^2 h}{dx^2} \right)}$$

Stable Solution:

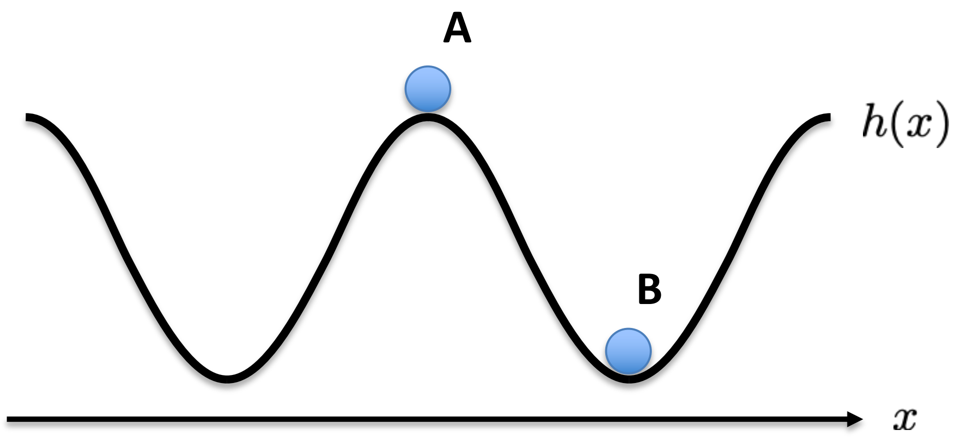
$$\left(\frac{d^2 h}{dx^2} \right)_{eq} > 0 \qquad \delta x = A \sin(\omega t + \phi) \qquad \omega = \sqrt{g \left(\frac{d^2 h}{dx^2} \right)}$$

The Nature of Instability

This problem can also be understood in terms of energetics:

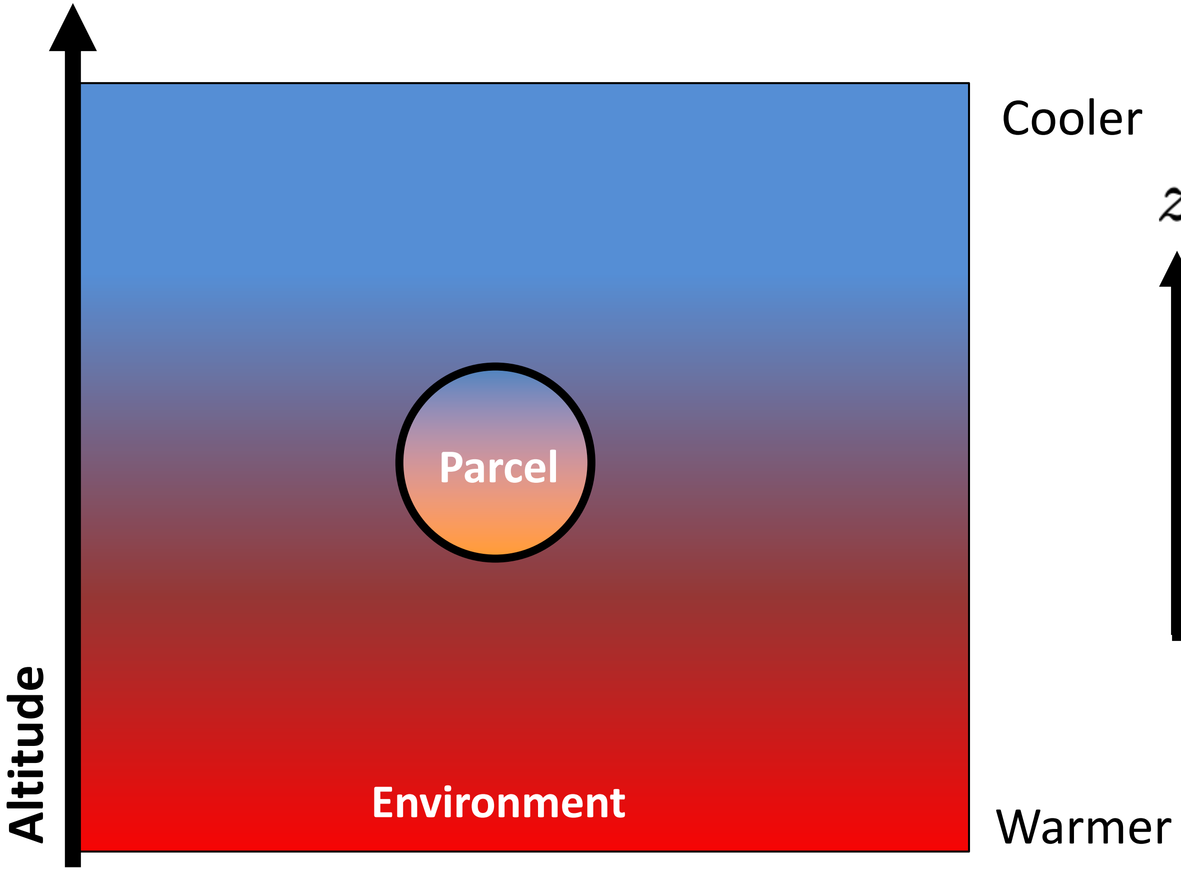
- A ball in position A converts **potential energy** into **kinetic energy** as it moves down the hill (hence as it falls it accelerates further)

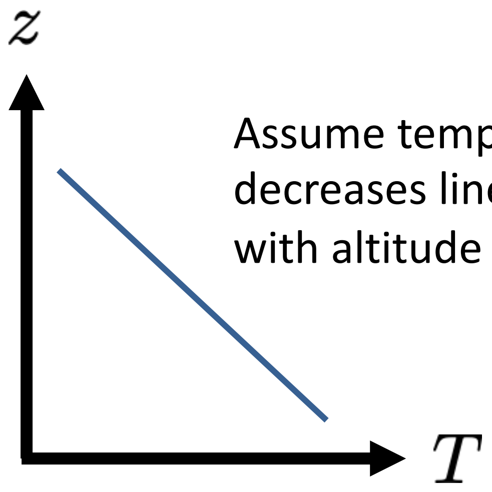
- A ball in position B must convert **kinetic energy** into **potential energy** as it moves up the hill, and so it slows as it ascends the hill.



Dry Atmospheric Convection

Dry Atmospheric Convection

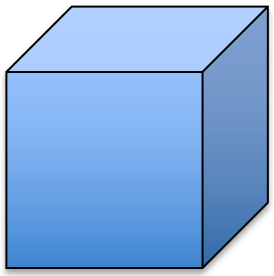




Assume temperature decreases linearly with altitude

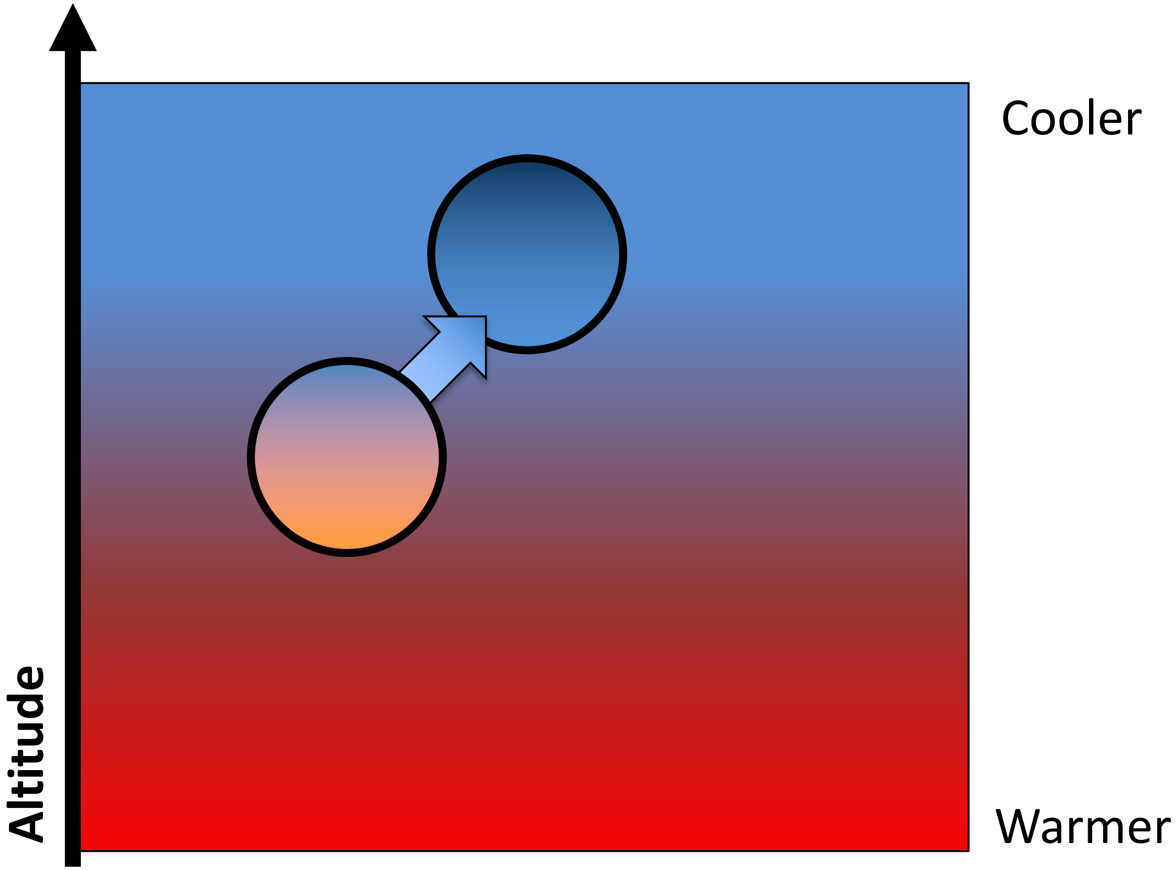
Dry Atmospheric Convection

The Parcel Method



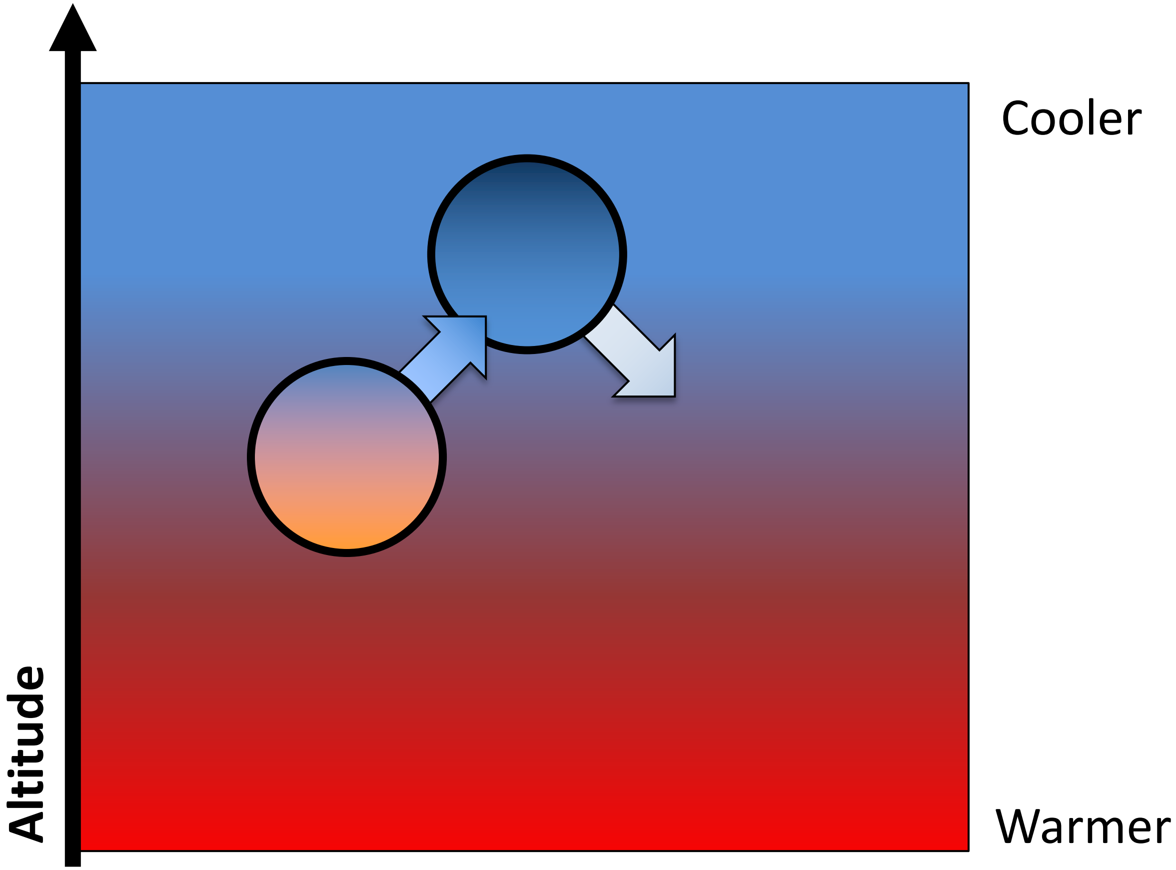
- Displace this parcel – move it up and down.
- Assume that the pressure adjusts instantaneously; that is, the parcel assumes the pressure of the altitude to which it is displaced.
- (for dry convection) As the parcel moves its temperature will change according to the adiabatic lapse rate. That is, the motion is without the addition or subtraction of energy ($J = 0$ in thermodynamic equation)

Dry Atmospheric Convection



If the parcel moves up and finds itself cooler than the environment then what will happen?

Dry Atmospheric Convection

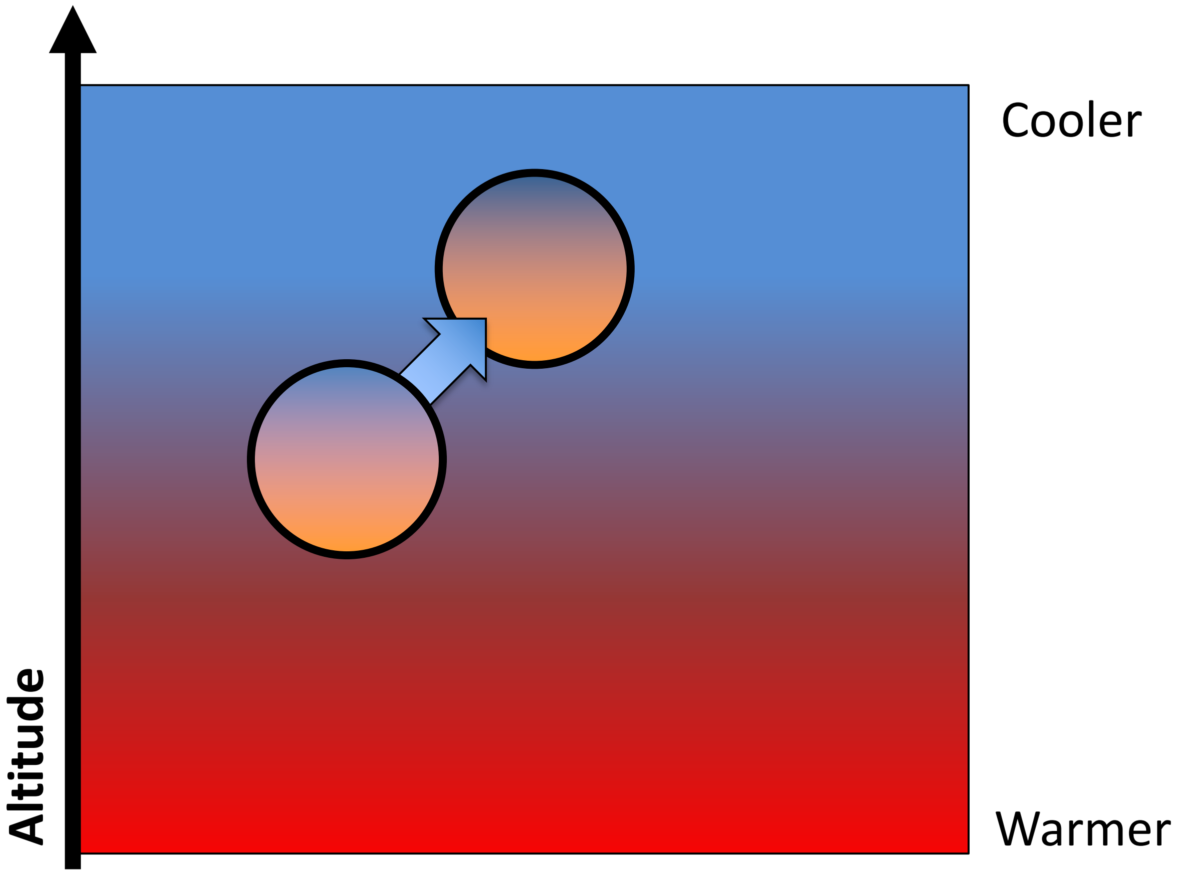


If the parcel moves up and finds itself cooler than the environment then what will happen?

Because the air parcel is **cooler** than the surrounding environment (at the same pressure), it is also **more dense** and so **will sink**.

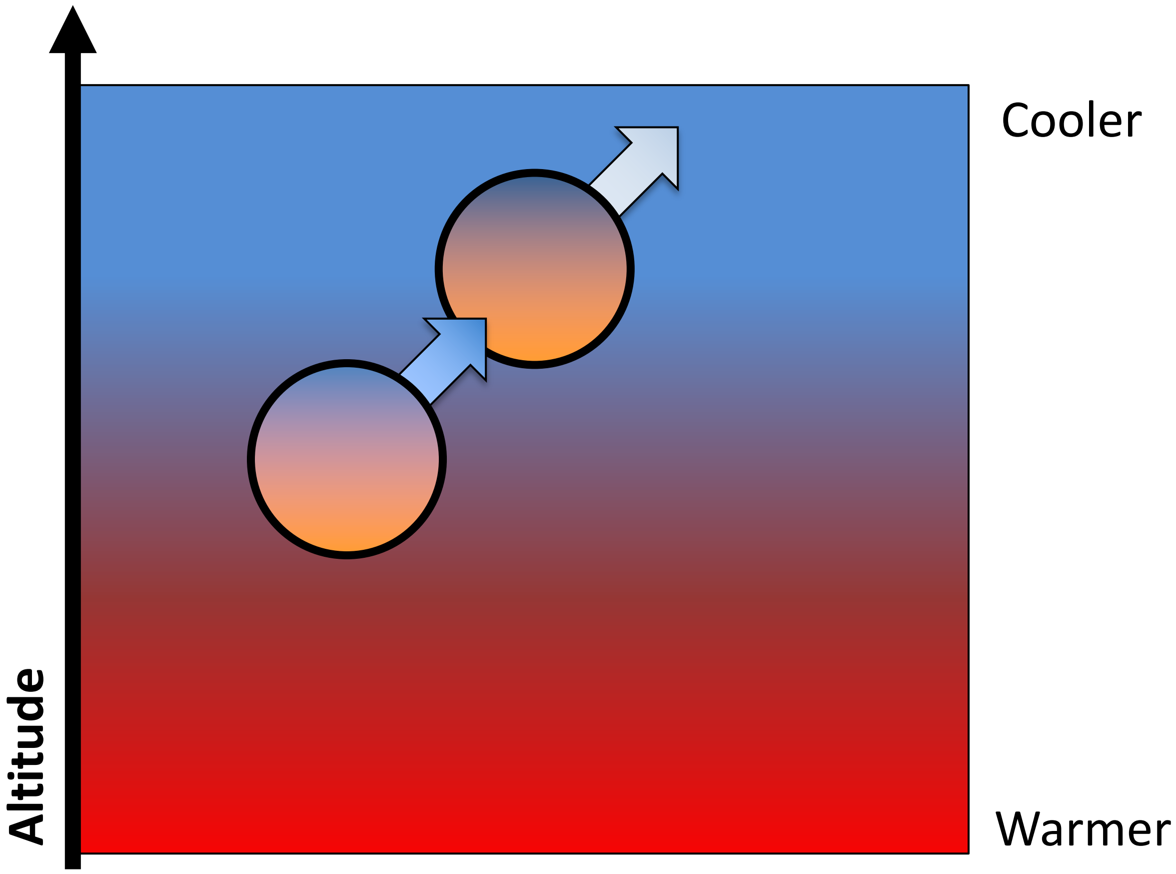
This is an example of **stability** – a controlled perturbation.

Dry Atmospheric Convection



If the parcel moves up and finds itself warmer than the environment then what will happen?

Dry Atmospheric Convection



If the parcel moves up and finds itself warmer than the environment then what will happen?

Because the air parcel is **warmer** than the surrounding environment (at the same pressure), it is also **less dense** and so will **continue to rise**.

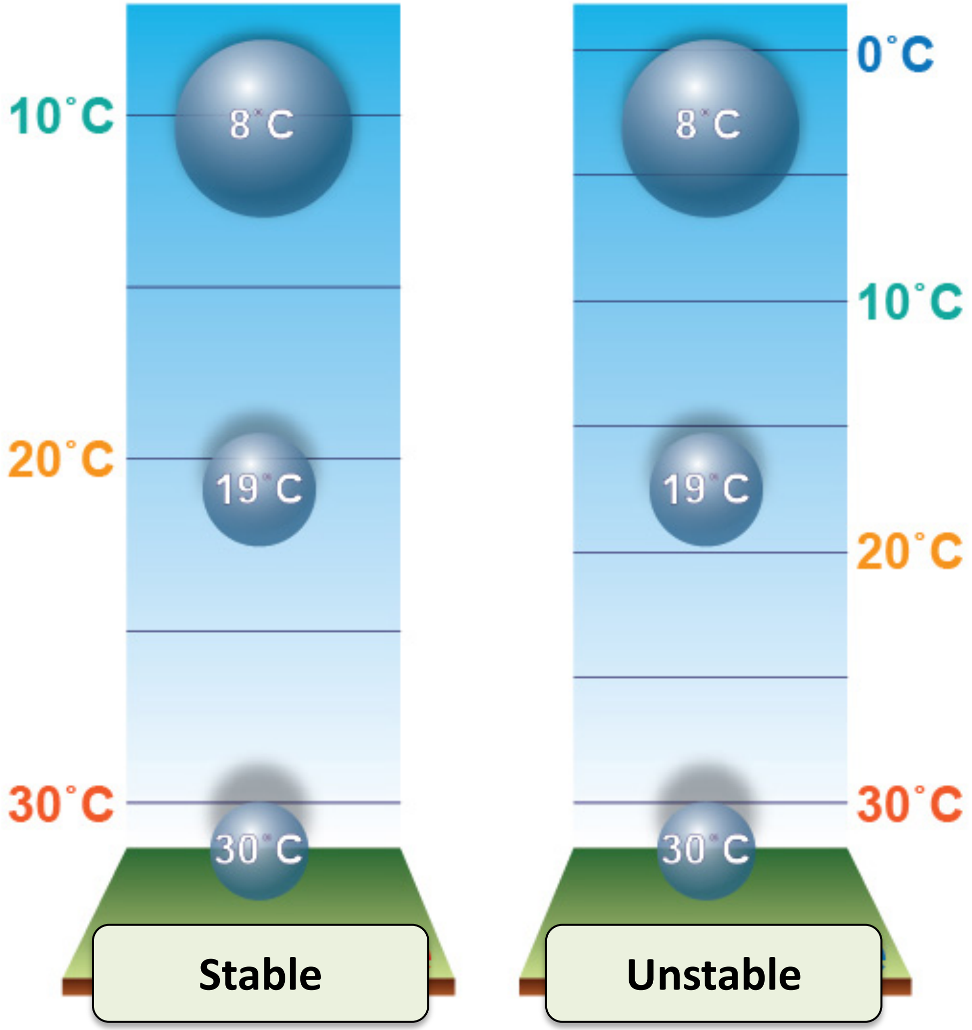
This is an example of **instability** – a growing perturbation.

Dry Atmospheric Convection

In summary, the rate of decrease of temperature with altitude (the lapse rate) determines the stability of the environment.

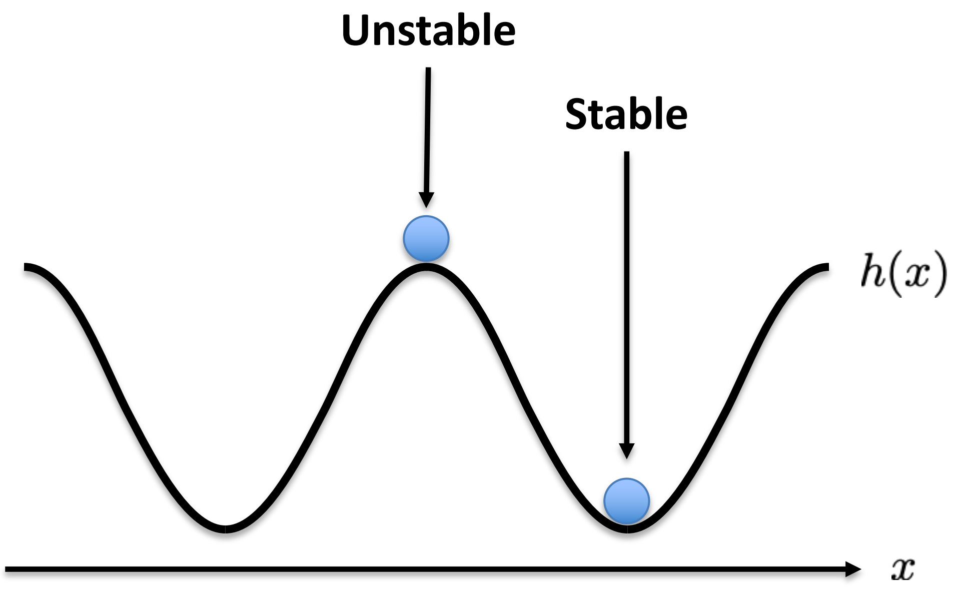
Under stable conditions temperatures decrease slowly or increase with altitude.

Under unstable conditions temperatures decrease rapidly with altitude.



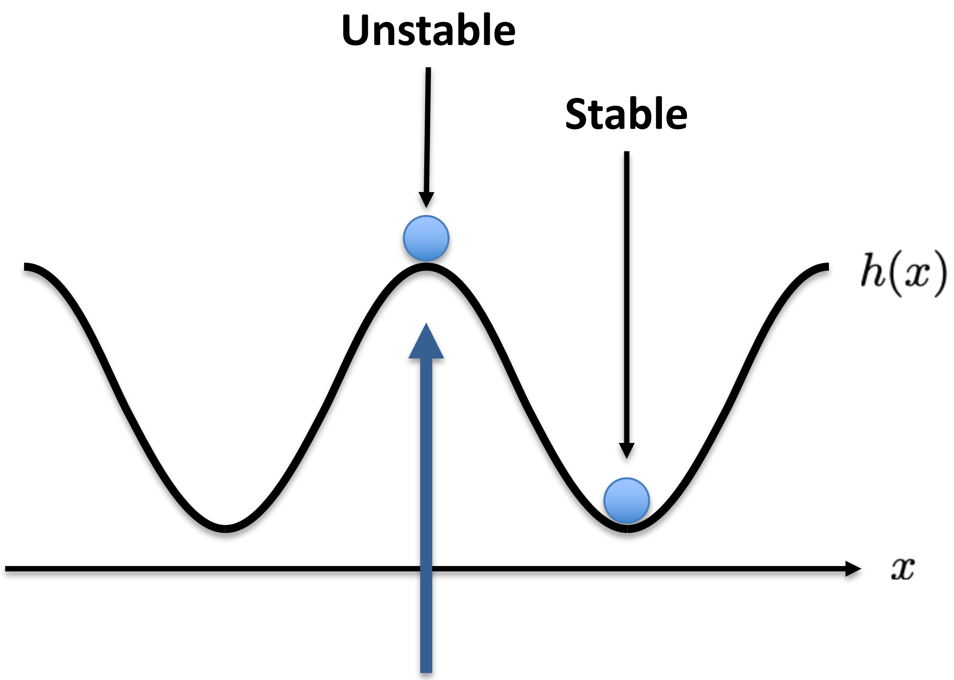
https://www.weather.gov/jetstream/parcels

Dry Atmospheric Convection



Definition: Under **unstable atmospheric conditions** small perturbations in the position of air parcels grow with time. Under **stable atmospheric conditions** small perturbations are controlled, i.e. air tends to resist movement.

Dry Atmospheric Convection



Surface warming builds up instability by developing warmer, less dense fluid underneath cooler, more dense fluid. We will now examine these conditions quantitatively.

Dry Atmospheric Convection

$$T = T_s - \Gamma z \qquad \Gamma = -\frac{\partial T}{\partial z}$$

Temperature of the atmosphere as a function of z and environmental lapse rate.

So if we go from z to $z + \Delta z$, then the change in T of the environment is

$$\begin{aligned} \Delta T &= [T_s - \Gamma(z + \Delta z)] - [T_s - \Gamma z] \\ &= -\Gamma \Delta z \end{aligned}$$

Under consideration of T changing with a constant linear slope (or lapse rate).

Dry Atmospheric Convection

So if we go from z to $z + \Delta z$, then the change in T of the parcel is

$$\begin{aligned}\Delta T_{\text{parcel}} &= T_{\text{parcel}}(z + \Delta z) - T_{\text{parcel}}(z) \\ &= \left(T_{\text{parcel}}(z) - \Gamma_d \Delta z\right) - T_{\text{parcel}}(z) \\ &= -\Gamma_d \Delta z\end{aligned}$$

$$\Gamma_d = \frac{g}{c_p} \equiv \text{adiabatic lapse rate}$$

Temperature of parcel changes with the dry adiabatic lapse rate.

Dry Atmospheric Convection

Stable: Temperature of parcel cooler than environment.

$$T_{\text{parcel}} < T_{\text{env}}$$

$$\Delta T_{\text{parcel}} < \Delta T_{\text{env}}$$

$$-\Gamma_d \Delta z < -\Gamma \Delta z$$

$$\Rightarrow \quad \boxed{\Gamma < \Gamma_d}$$

Dry Atmospheric Convection

Unstable: Temperature of parcel warmer than environment.

$$T_{\text{parcel}} > T_{\text{env}}$$

$$\Delta T_{\text{parcel}} > \Delta T_{\text{env}}$$

$$-\Gamma_d \Delta z > -\Gamma \Delta z$$

$$\Rightarrow \quad \boxed{\Gamma > \Gamma_d}$$

Dry Atmospheric Convection

Stability criteria from physical argument:

$$\Gamma_d < \Gamma \quad \textbf{unstable}$$

$$\Gamma_d = \Gamma \quad \textbf{neutral}$$

$$\Gamma_d > \Gamma \quad \textbf{stable}$$

Adiabatic lapse rate

Environmental lapse rate

Key point: A compressible atmosphere is unstable if temperature decreases with height faster than the adiabatic lapse rate.

Dry Static Stability

Environment is in hydrostatic balance, no acceleration:

$$0 = -\frac{1}{\rho_{env}}\frac{\partial p_{env}}{\partial z} - g$$

But our parcel experiences an acceleration:

$$\frac{Dw}{Dt} = \frac{D^2 z}{Dt^2} = -\frac{1}{\rho_{parcel}}\frac{\partial p_{env}}{\partial z} - g$$

Assume instantaneous adjustment of parcel pressure

Dry Static Stability

But our parcel experiences an acceleration:

$$\frac{Dw}{Dt} = \frac{D^2 z}{Dt^2} = \frac{g\rho_{env}}{\rho_{parcel}} - g$$

$$= g\left(\frac{\rho_{env} - \rho_{parcel}}{\rho_{parcel}}\right)$$

Ideal gas law:

$$\rho_{env} = \frac{p_{env}}{RT_{env}} \qquad \rho_{parcel} = \frac{p_{env}}{RT_{parcel}}$$

$$\Rightarrow \quad \frac{D^2 z}{Dt^2} = g\left(\frac{T_{parcel} - T_{env}}{T_{env}}\right)$$

Dry Static Stability

$$\frac{D^2 z}{Dt^2} = g\left(\frac{T_{parcel} - T_{env}}{T_{env}}\right)$$

$$T_{parcel} = T(z_0) - \Gamma_d(z - z_0)$$

$$T_{env} = T(z_0) - \Gamma(z - z_0)$$

$$\Rightarrow \quad \frac{D^2 z}{Dt^2} = \frac{g}{T(z_0) - \Gamma(z - z_0)}(\Gamma - \Gamma_d)(z - z_0)$$

Binomial expansion:

$$\frac{1}{T(z_0) - \Gamma(z - z_0)} = \frac{1}{T(z_0)} \frac{1}{1 - \frac{\Gamma}{T(z_0)}(z - z_0)}$$

$$\approx \frac{1}{T(z_0)}\left(1 + \frac{\Gamma(z - z_0)}{T(z_0)}\right)$$

Dry Static Stability

$$\frac{D^2 z}{Dt^2} = \frac{g}{T(z_0)}(\Gamma - \Gamma_d)(z - z_0) + \frac{g\Gamma}{T(z_0)^2}(z - z_0)^2$$

For small displacements ignore quadratic terms:

$$\frac{D^2 z}{Dt^2} \approx \frac{g}{T(z_0)}(\Gamma - \Gamma_d)(z - z_0)$$

Define $\Delta z = z - z_0$

$$\frac{D^2 \Delta z}{Dt^2} + \frac{g}{T(z_0)}(\Gamma_d - \Gamma)\Delta z = 0$$

Dry Static Stability

$$\frac{D^2 \Delta z}{Dt^2} + \frac{g}{T(z_0)}(\Gamma_d - \Gamma)\Delta z = 0$$

This is an ordinary differential equation in the variable Δz. Do you recognize this equation and its solutions?

Three cases:

$$\frac{g(\Gamma_d - \Gamma)}{T(z_0)} > 0$$

$$\frac{g(\Gamma_d - \Gamma)}{T(z_0)} = 0$$

$$\frac{g(\Gamma_d - \Gamma)}{T(z_0)} < 0$$

Correspond to stable, neutral and unstable atmospheres.

Stable Conditions

$$\frac{D^2\Delta z}{Dt^2} + \frac{g}{T(z_0)}(\Gamma_d - \Gamma)\Delta z = 0$$

$$\frac{g(\Gamma_d - \Gamma)}{T(z_0)} > 0$$

Stable atmosphere

Definition: The **Brunt-Väisälä Frequency** is given by

$$\mathcal{N} = \sqrt{\frac{g(\Gamma_d - \Gamma)}{T(z_0)}}$$

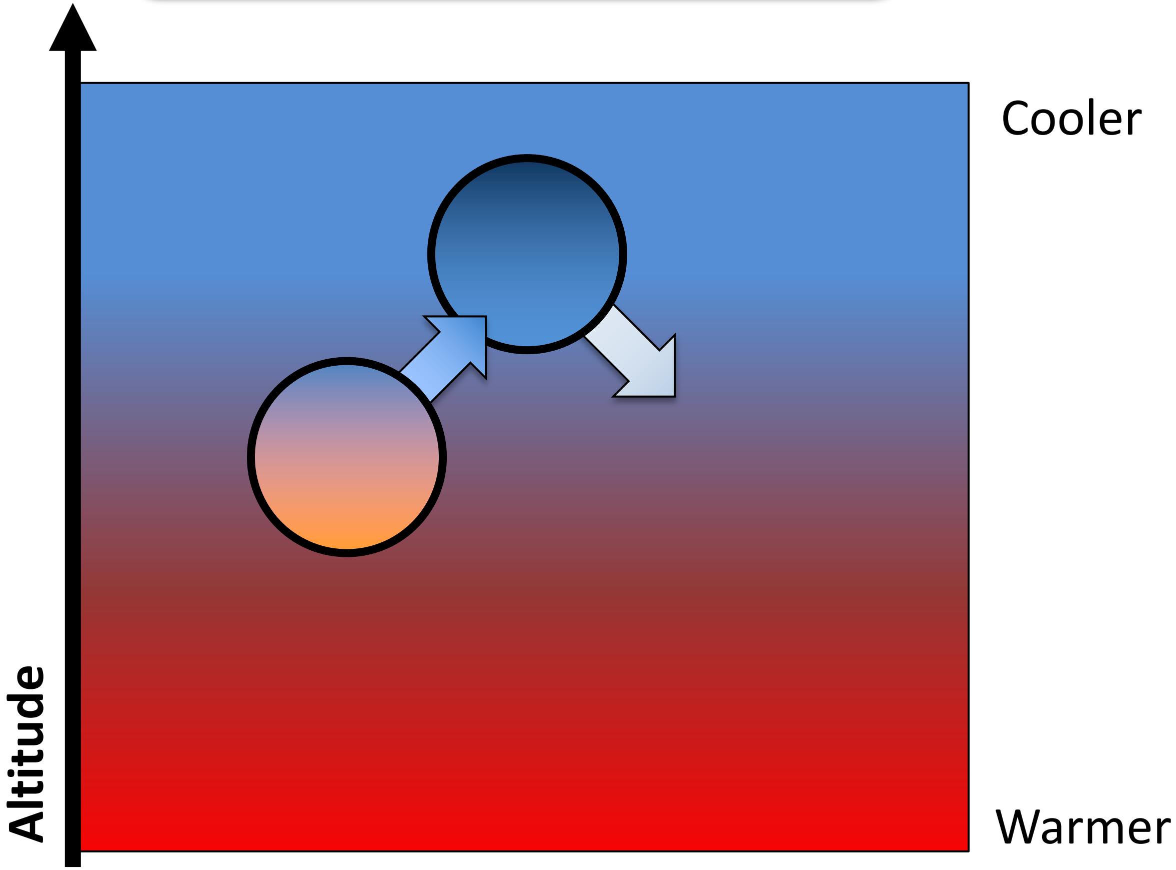
It represents the oscillatory frequency of a dry air parcel under stable environmental conditions (units of inverse seconds (Hz)).

$$\Delta z = \Delta z_0 \sin\left(\mathcal{N} t + \phi\right)$$

Oscillatory solutions (magnitude of oscillation depends on initial velocity)

Stable Conditions

$$\mathcal{N}^2 = \frac{g(\Gamma_d - \Gamma)}{T(z_0)} > 0$$



If the parcel moves up and finds itself cooler than the environment then what will happen?

Because the air parcel is **cooler** than the surrounding environment (at the same pressure), it is also **more dense** and so **will sink**.

This is an example of **stability** – a controlled perturbation.

Stable Conditions

Figure: An example of a stable environment with oscillating air parcels.

Stable Conditions

Figure: Another example of a stable environment with a cloud train.

Stable Conditions

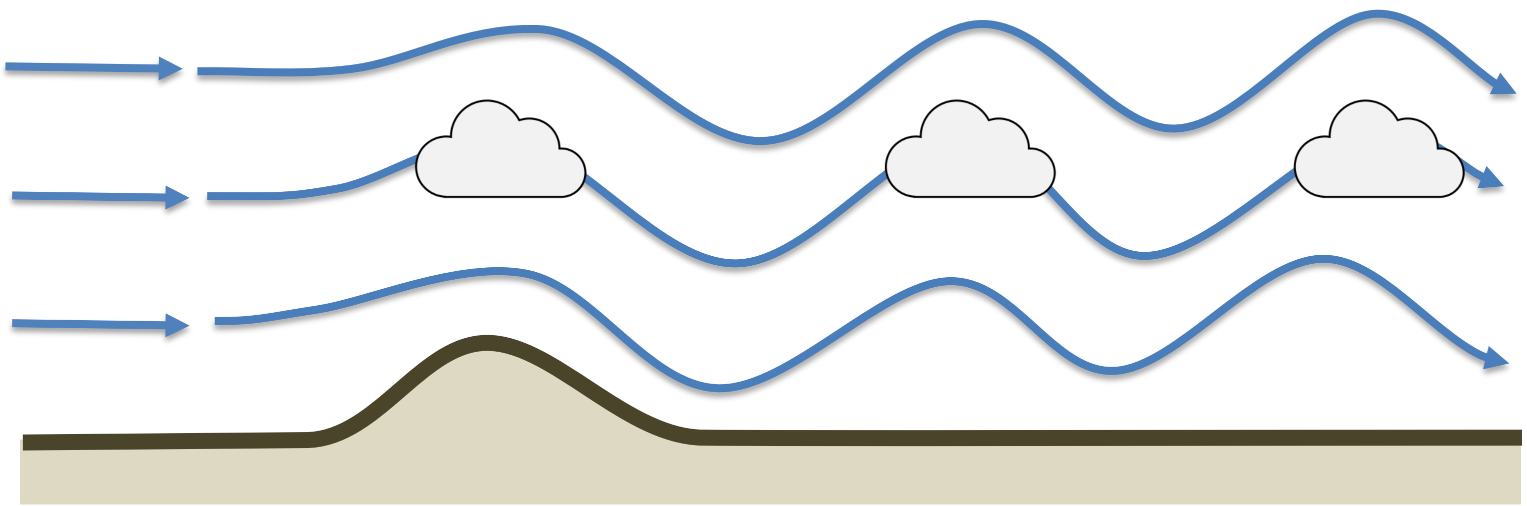


Figure: A schematic diagram illustrating the formation of **mountain lee waves**. These develop as air moves over a topographic feature in a stable environment, producing a train of downstream waves.

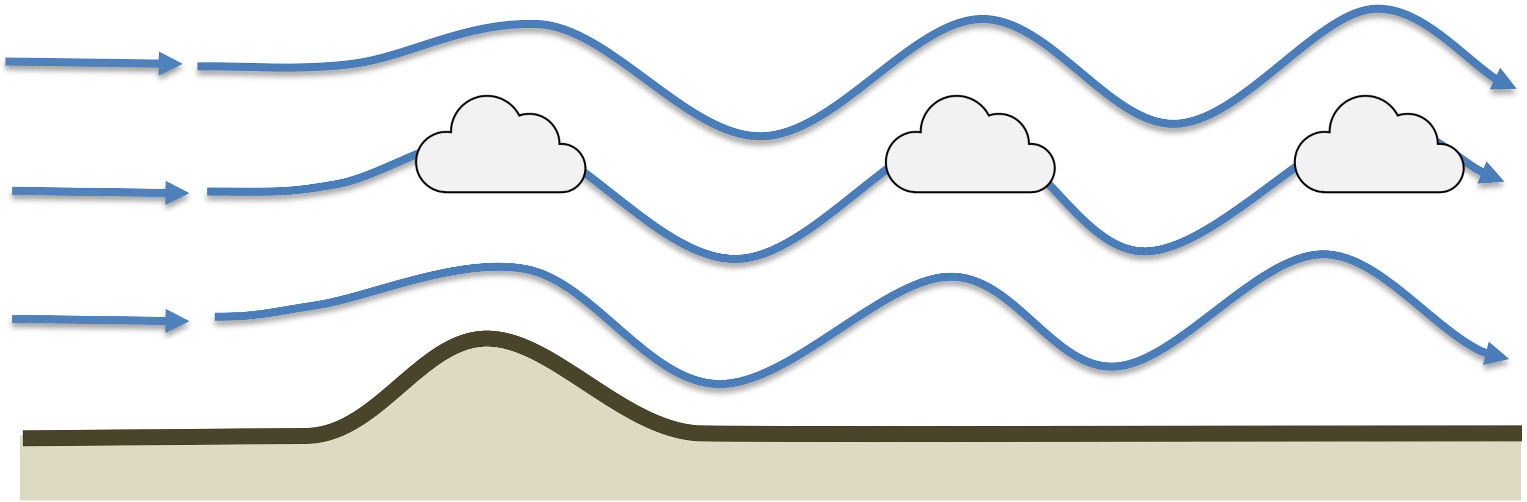
Directly over the mountain, a distinct cloud type known as a **lenticular** ("lens-like") cloud is frequently produced.

Downstream and aloft, cloud bands may mark parts of the **wave train** in which air has been uplifted (and thus cooled to saturation).

Stable Conditions

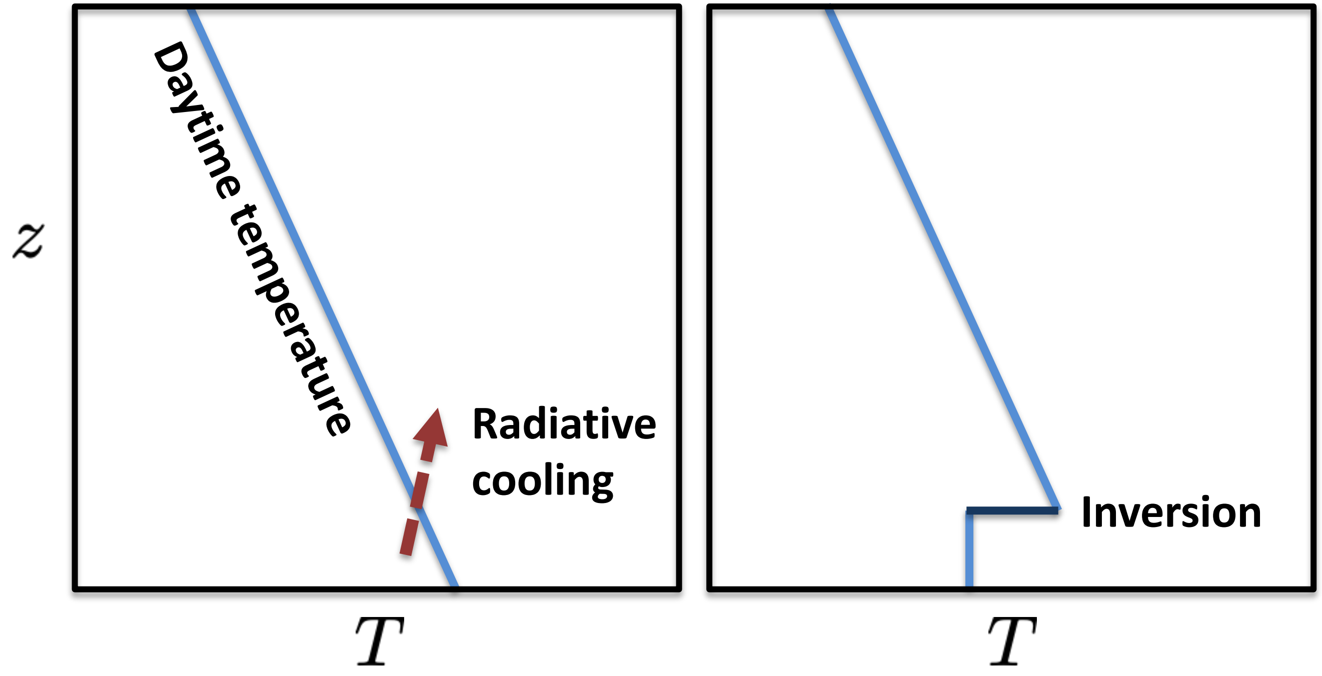
Figure: A **lenticular** ("lens-like") cloud.

Stable Conditions



Test your knowledge: Under typical tropical tropospheric conditions, θ_1 = 300K at the surface and θ_2 = 340K at the troposphere (height 10km). If the wind is blowing at 20 m/s horizontally, what is the approximate distance between cloud decks that have formed in the lee of a mountain range?

Stable Conditions (Temperature Inversions)



Definition: Low-level **temperature inversions** correspond to a reversal of the normal behavior of temperature within the troposphere – namely, temperature that increases with altitude.

These are commonly produced during calm winter nights from radiative cooling of the surface. Because temperature increases with altitude air is unable to rise through the inversion.

Temperature Inversions

Figure: A temperature inversion in the Lake District, England, forms clouds at low level beneath clear skies.

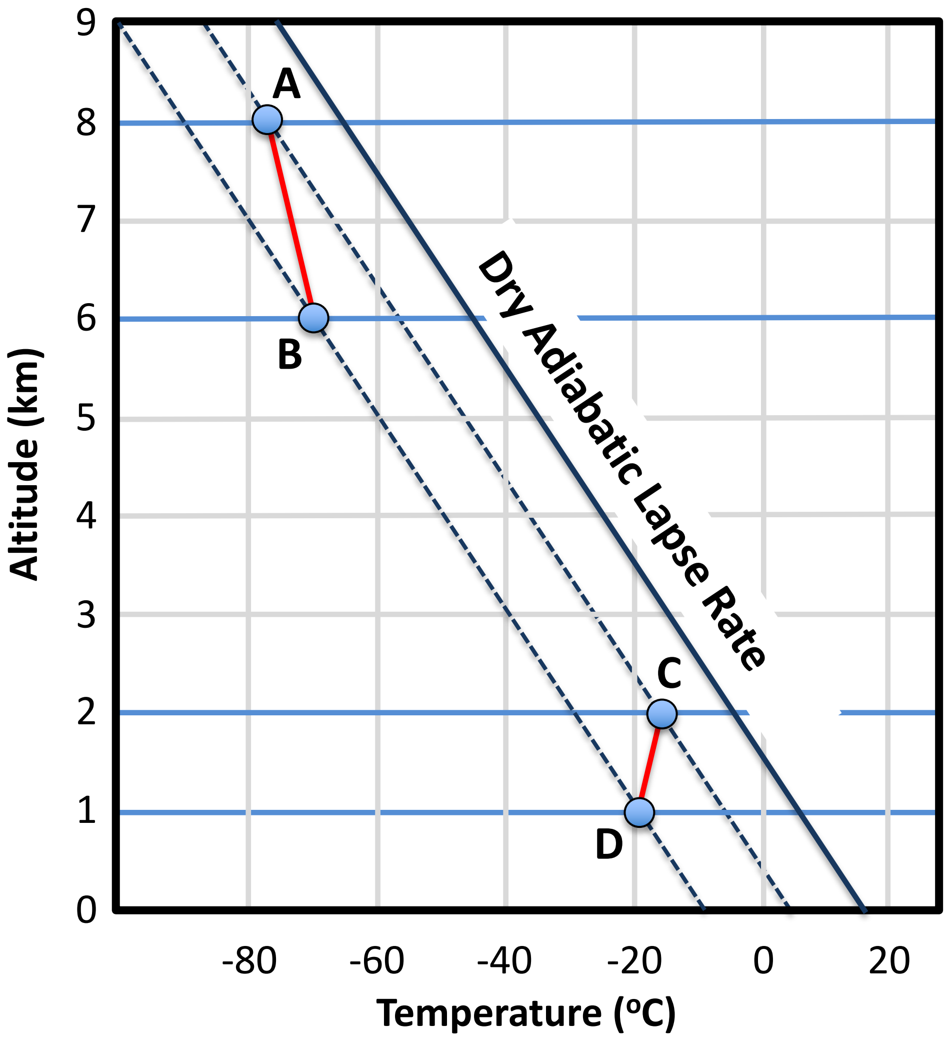
Temperature Inversions

Figure: Smog trapped over the city of Almaty, Kazakhstan during a temperature inversion.

Temperature Inversions

Figure: A **trade inversion** can also be created by descent and adiabatic warming.

When the layer from 6-8 kilometers (designated A-B) descends dry adiabatically along the dashed lines, it compresses and warms. The result is the near-surface inversion C-D.



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Neutral Conditions

$$\frac{D^2 \Delta z}{Dt^2} + \frac{g}{T(z_0)}(\Gamma_d - \Gamma)\Delta z = 0$$

$$\frac{g(\Gamma_d - \Gamma)}{T(z_0)} = 0$$ **Neutral atmosphere**

$$\Delta z = u_0 t$$

Parcel does not experience acceleration; travels at initial velocity.

Unstable Conditions

$$\frac{D^2 \Delta z}{Dt^2} + \frac{g}{T(z_0)}(\Gamma_d - \Gamma)\Delta z = 0$$

$$\frac{g(\Gamma_d - \Gamma)}{T(z_0)} < 0$$

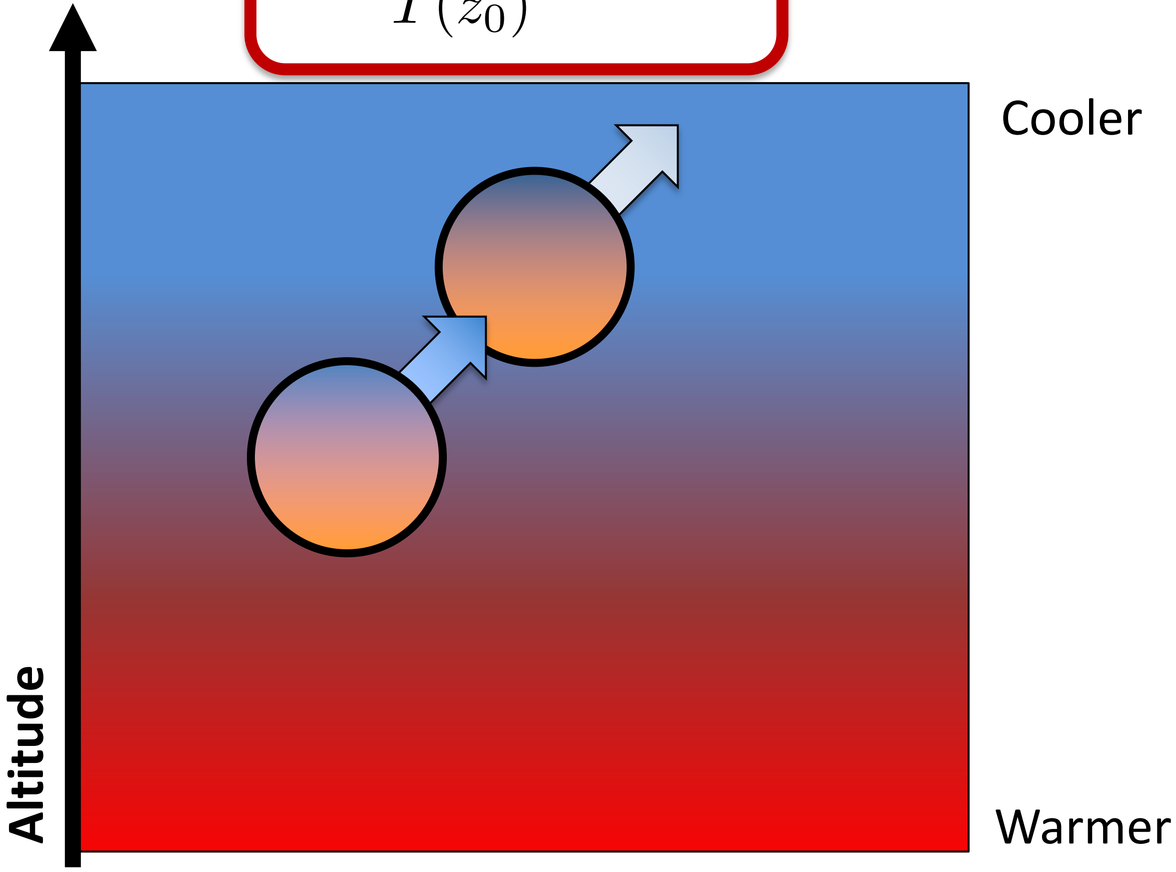
Unstable atmosphere

$$\Delta z = A \exp\left(\frac{g(\Gamma - \Gamma_d)}{T(z_0)} t\right) + B \exp\left(-\frac{g(\Gamma - \Gamma_d)}{T(z_0)} t\right)$$

Exponential solutions (at least one of these terms will grow without bound).

Unstable Conditions

$$\frac{g(\Gamma_d - \Gamma)}{T(z_0)} < 0$$



If the parcel moves up and finds itself warmer than the environment then what will happen?

Because the air parcel is **warmer** than the surrounding environment (at the same pressure), it is also **less dense** and so will **continue to rise**.

This is an example of **instability** – a growing perturbation.

Unstable Conditions

Figure: An anvil cloud over Africa produced by rising air in an unstable environment. The anvil top is produced when the rising air reaches the tropopause.

Within the stratosphere the environmental temperature increases with altitude, which is strongly stable.

Source: Image Science & Analysis Laboratory, NASA Johnson Space Center

Unstable Conditions

Figure: Thunderheads loom over the US midwest 8/19/2003.

Source: Image Science & Analysis Laboratory, NASA Johnson Space Center

Static Stability

Brunt-Väisälä Frequency

$$\mathcal{N}^2 = g\frac{\partial(\ln\theta)}{\partial z} = \frac{g}{\theta}\frac{\partial\theta}{\partial z}$$

Exercise: Show this definition of Brunt-Vaisala frequency is equivalent to the previous definition.

$$\frac{\partial\theta}{\partial z} > 0 \quad \text{statically stable}$$

$$\frac{\partial\theta}{\partial z} = 0 \quad \text{statically neutral}$$

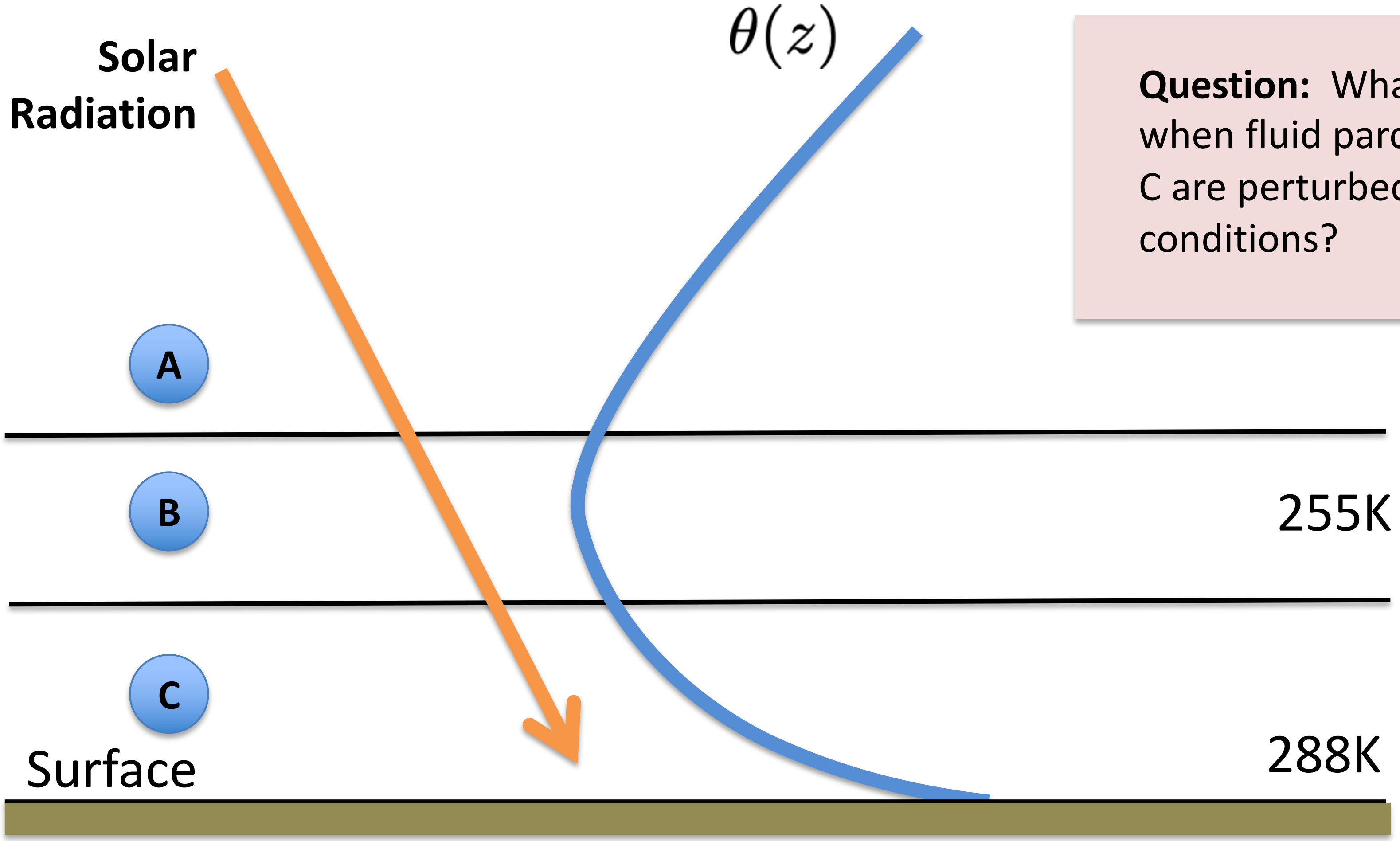
$$\frac{\partial\theta}{\partial z} < 0 \quad \text{statically unstable}$$

Static Stability

Two ways of determining dry static stability:

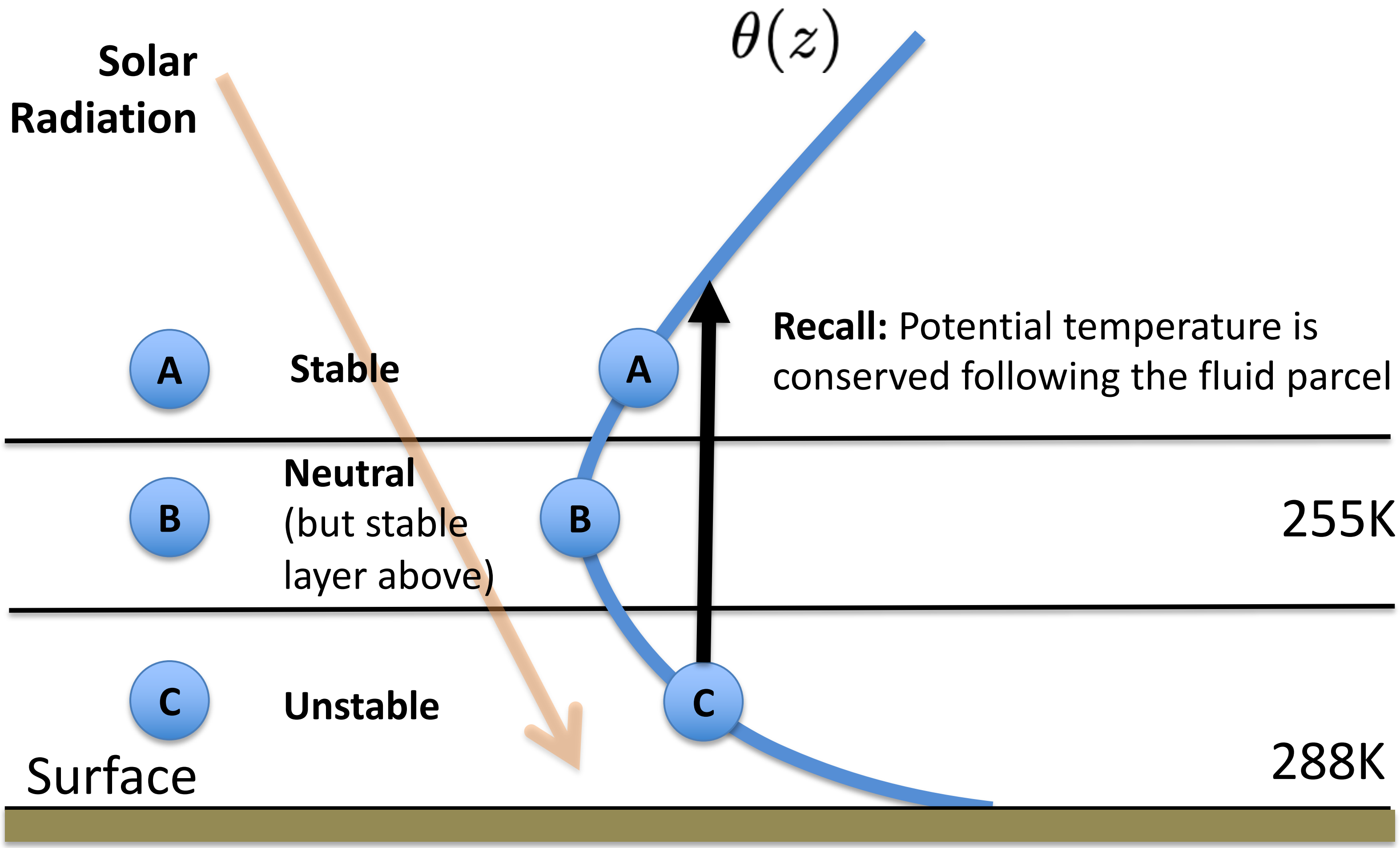
via environmental lapse rate	via potential temperature	
$\Gamma < \Gamma_d$	$\dfrac{\partial \theta}{\partial z} > 0$	(statically stable)
$\Gamma = \Gamma_d$	$\dfrac{\partial \theta}{\partial z} = 0$	(statically neutral)
$\Gamma > \Gamma_d$	$\dfrac{\partial \theta}{\partial z} < 0$	(statically unstable)

Static Stability

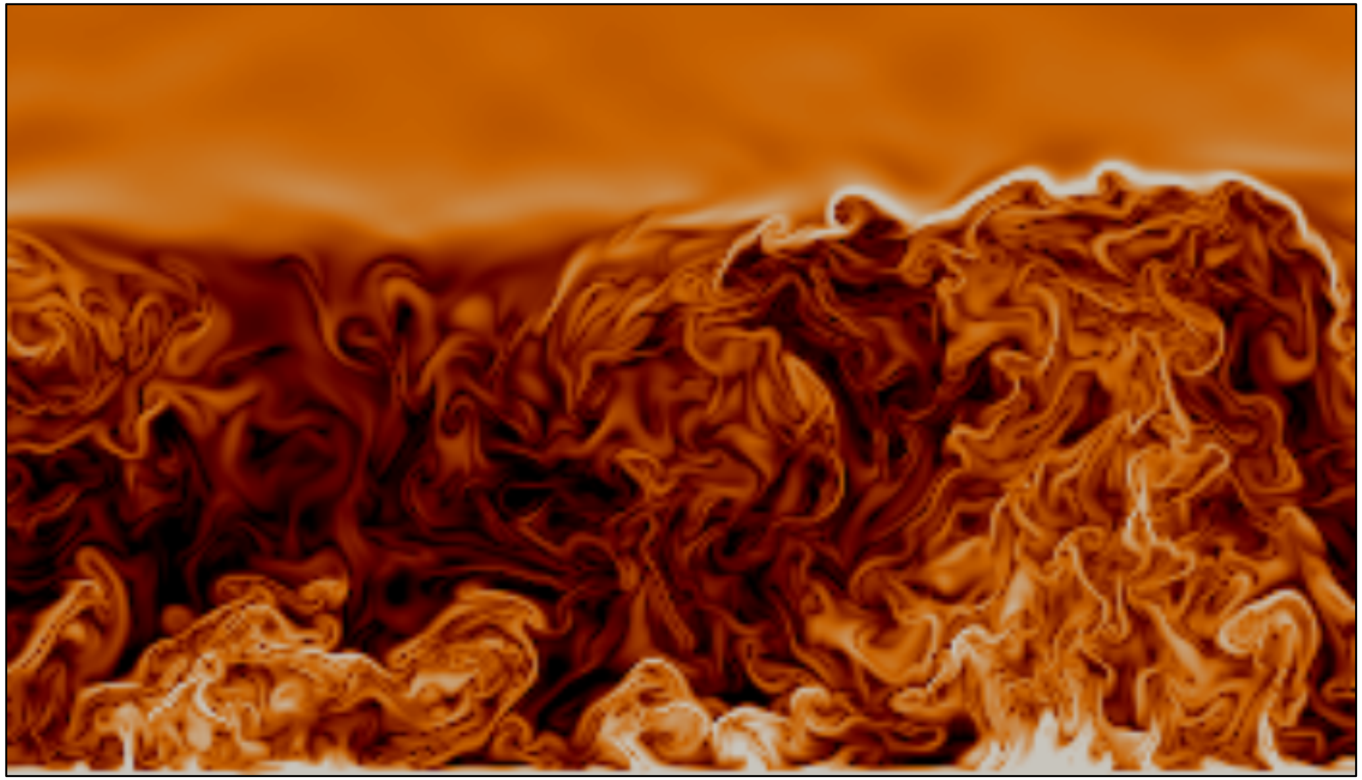


Question: What happens when fluid parcels A, B and C are perturbed under dry conditions?

Static Stability



Convection



Free Atmosphere / Stratified Layer

Inversion

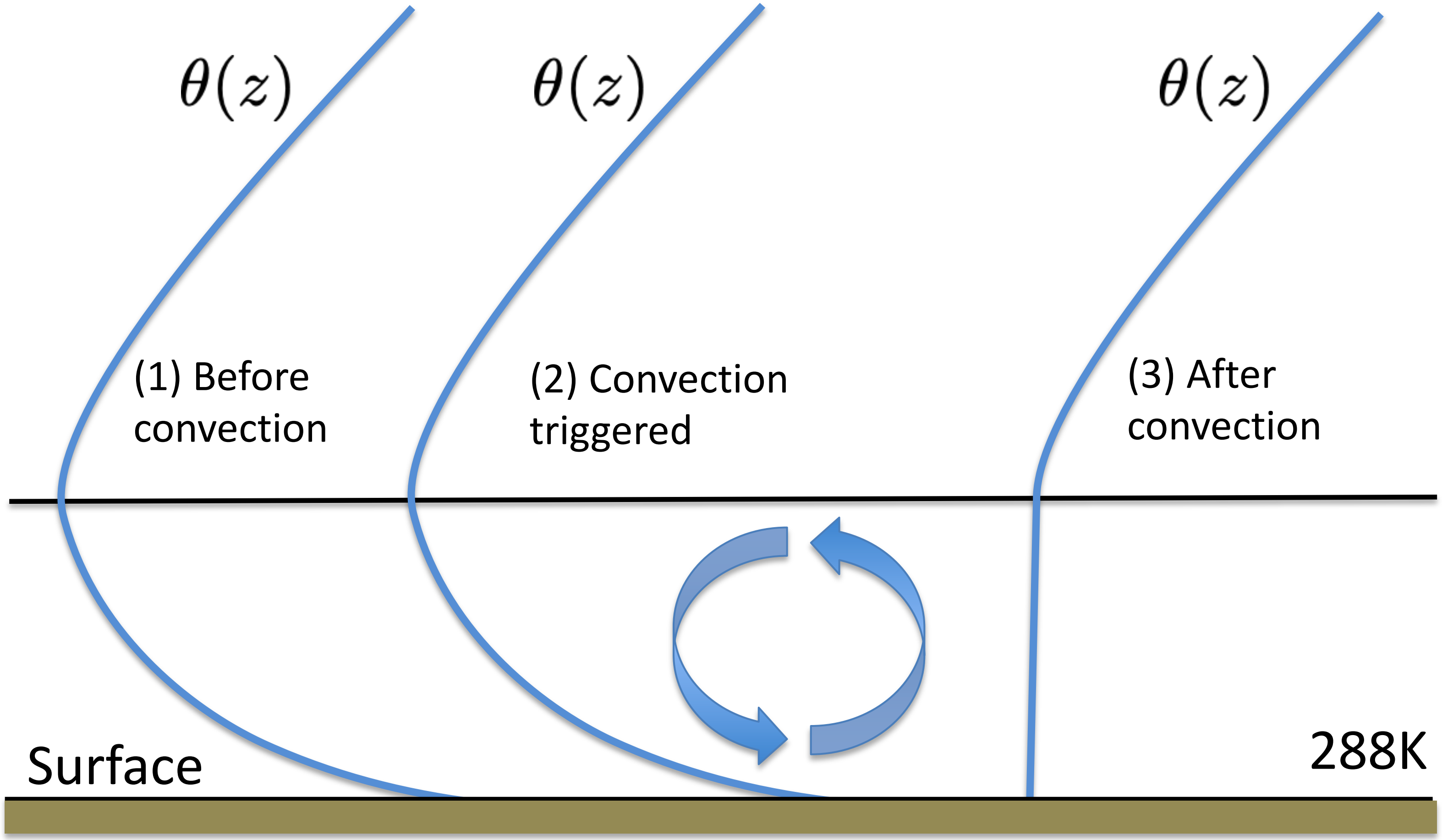
Turbulent Boundary Layer

Figure: A snapshot of a convecting boundary layer. Note the undulations on the inversion caused by convection overshooting the well-mixed layer below into the stratified layer above.

https://blogs.egu.eu/divisions/as/tag/rayleigh-benard-convection/

Convection

Key point: Convection is an inherently stabilizing process, since mixing causes potential temperature to stabilize.



Convection

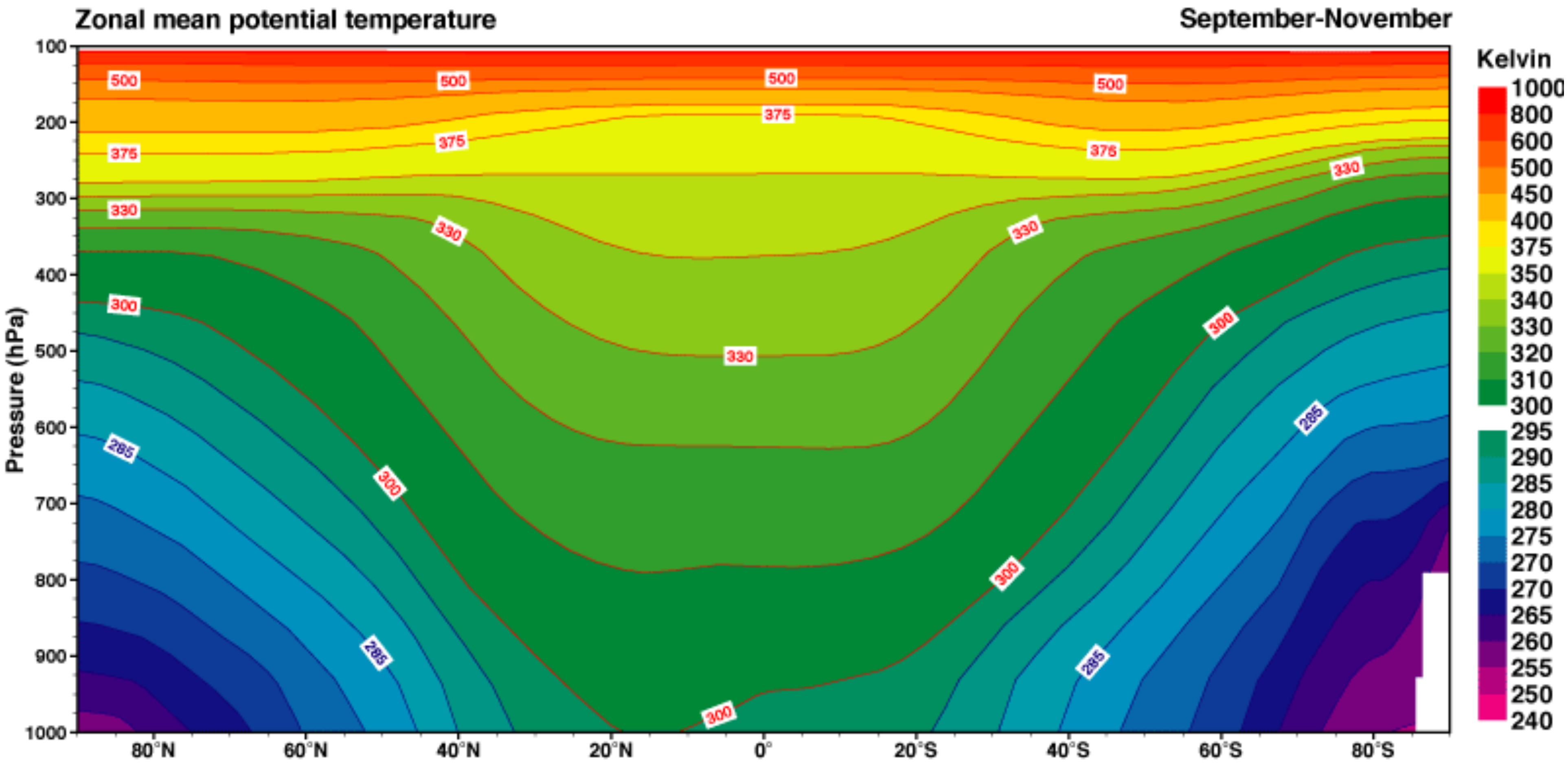


Figure: Zonally averaged potential temperature (September-November) from ERA-40 reanalysis data. Note the relatively mild increase in potential temperature in the equatorial region.

ATM 241 Climate Dynamics
Lecture 4b
Dry Convection

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Thank You!