ATM 241 Climate Dynamics
Lecture 2
Energy and the Earth System

Paul A. Ullrich
paullrich@ucdavis.edu

Marshall & Plumb
Ch. 2
### Definitions

- Temperature
- Conduction
- Convection
- Radiation
- Energy balance
- Black body
- Solar constant
- Insolation
- Albedo

- Planetary albedo
- Emission temperature
- Greenhouse effect
- Greenhouse gases
- Climate sensitivity
- Carbon dioxide sensitivity
- Positive climate feedback
- Negative climate feedback
Questions

• Why is energy balance a good approximation for the Earth system?
• How does insolation change as a function of latitude and season?
• How is insolation connected with the local climatology?
• Why is the top of the atmosphere insolation different from surface insolation?
• What are the different pathways for radiation that reaches the Earth?
• What simple models can be used to approximate radiative exchange?
• Why is climate sensitivity hard to estimate?
Definition: Kinetic energy (KE) is a measure of the energy of a substance due to motion. At a macroscopic scale, temperature is a measure of the average kinetic energy of molecules in a particular substance (such as a parcel of air).

Although an air parcel may appear stationary, the molecules that make up that air parcel are moving around and interacting with one another.

Cold air is made up of molecules that are moving relatively slowly (carrying less kinetic energy).

Hot air is made up of molecules that are moving relatively quickly (carrying more kinetic energy).
Heat can be transferred via:

- **Conduction**: Heat transfer due to contact. Molecules of a warm object bang against molecules of a cold object, causing “cold” molecules to move faster and “warm” molecules to move slower.

- **Convection**: Heat transfer due to movement. A balloon full of warm air can move into a region of cold air and increase the average air temperature.

- **Radiation**: Heat transfer due to photons (small bundles of radiated energy). When these photons hit another molecule, kinetic energy is transferred.
Conservation of Energy is a fundamental rule of nature, requiring that energy be neither created nor destroyed.

Change in Energy = Energy Received - Energy Emitted

If a system is in a state of energy balance then the total energy within the system must be unchanging, which implies:

Energy Received = Energy Emitted
The principles that govern radiation state that the amount of energy emitted from an object increases and decreases with the energy of that object.

\[
\text{Energy Emitted} \propto \text{Temperature}^4
\]

Thus the more energy an object has, the more energy it will radiate.

\[
\text{Change in Energy} = \text{Energy Received} - \text{Energy Emitted}
\]

If an object is receiving more energy than it emits, then it will warm until its emission equals the energy received. If it is receiving less energy than it emits, then it will cool until its emission equals the energy received.
Global Energy Balance

Since we know that

1. the Earth has had a very long time to equilibrate its temperature, and
2. the amount of energy received by the Earth does not change significantly on time scales of millions of years,

it is a natural consequence that the Earth system is, to a close approximation, in a state of energy balance:

\[
\text{Energy Received by the Earth} = \text{Energy Emitted by the Earth}
\]

Note that energy balance is not affected by human activities. Even under global warming, in equilibrium energy emitted must equal energy received.
To maintain its global energy balance, the Earth must radiate energy at the same rate that it receives it from the sun.
**Definition:** A black body is an idealized physical body that absorbs all incident electromagnetic radiation. A black body in thermal equilibrium (constant temperature) emits blackbody radiation in accordance with Planck’s law.

- A black body is an **ideal emitter**: it emits as much or more energy at every frequency / wavelength than any other body at the same temperature.

- A black body is a **diffuse emitter**: the energy is emitted isotropically, independent of direction.
Black Body Radiation

**Definition:** A **black body** is an idealized physical body that absorbs all incident electromagnetic radiation. A black body in thermal equilibrium (constant temperature) emits blackbody radiation in accordance with Planck’s law.

**Planck’s law:**

\[
B_\lambda(T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp \left( \frac{hc}{\lambda k_B T} \right) - 1}
\]

This is analogous to a probability density function for radiation. Hence the total emission can be obtained by integrating over all wavelengths:

**Stefan-Boltzmann law:**

\[
j^* = \sigma T^4 \quad \text{Result in } W / m^2
\]

\[
\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} = 5.670400 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4}
\]
**Key Point:** The sun and the Earth can be approximately considered to be black bodies. Hence the type of radiation emitted by these objects can be determined from their temperatures.
Stefan-Boltzmann law: \( j^* = \sigma T^4 \) Result in Watts / meter\(^2\)

Stefan-Boltzmann constant: \( \sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} = 5.670400 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4} \)

Question: The sun’s photosphere can be found at a radius of approximately 700 000 km and has an average temperature of approximately 5770K. Using this information approximate the total amount of energy emitted from the sun.

\[ Q = \] W
The energy from the sun is approximately isotropic (emitted in all directions equally). With this in mind, the Earth only receives a tiny fraction of the sun’s radiation (although the amount received is approximately constant).

**Definition:** The solar constant is the rate at which solar energy is received by the Earth (per unit area).

**Approximate Earth-sun distance**

\[ R_E \approx 1.50 \times 10^{11} \text{m} \]

\[ S_0 = \frac{Q}{4\pi R_E^2} \]

\[ S_0 \approx 1367 \text{ W m}^{-2} \]
Global Energy Balance

**Question:** What is the total power incident onto the Earth from the sun?

Solar constant
\[ S_0 \approx 1367 \text{ W m}^{-2} \]

Radius of Earth
\[ a = 6371 \text{ km} \]

\[ P_{inc} = \text{ } \text{W} \]
Follow-up Question: If the Earth had no atmosphere, what is the (spatially averaged) instantaneous power per unit area received by the Earth?

Solar constant  \( S_0 \approx 1367 \text{ W m}^{-2} \)

Radius of Earth  \( a = 6371 \text{ km} \)

\[ P_{\text{avg}} \approx \_ \text{ W m}^{-2} \]
The curvature of the Earth’s surface means that the power due to incident solar radiation is not equal across the surface.

The amount of solar radiation received at a given point is proportional to the cosine of the great circle distance between that point and sub-solar point (point directly under the sun).

**Definition: Insolation** is a measure of the solar radiation energy received on a given surface area and recorded during a given time.
Calculating Instantaneous Insolation

\[ \delta \quad \text{Latitude of sub-solar point} \]
\[ h \quad \text{Hour angle (=0 at sub-solar point)} \]
\[ (\phi, \lambda) \quad \text{Latitude / longitude of observer} \]
\[ \Theta \quad \text{Great circle arc length} \]

*(Great circle arc length formula)*

\[ \cos \Theta = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h \]

\[ R_o \quad \text{Mean radius of Earth’s orbit} \quad R_E \quad \text{Current radius of Earth’s orbit} \]

*(Instantaneous Insolation)*

\[ Q(\phi, \lambda) = \begin{cases} 
0 & \text{if } \cos \Theta < 0, \\
S_o \frac{R_o^2}{R_E^2} \cos \Theta & \text{otherwise.}
\end{cases} \]
The latitude of the sub-solar point ($\delta$) can be calculated in terms of the Earth’s axial tilt ($\epsilon$) and the polar angle of the Earth’s orbit ($\theta$):

$$\sin \delta = \sin \epsilon \sin \theta$$

with axial tilt $\epsilon = 23.4398^\circ$
Calculating Daily Average Insolation

Averaging instantaneous insolation over a day then gives the **daily average insolation**:

\[
\bar{Q}_{\text{day}} = \frac{1}{2 \pi} \int_{-\pi}^{\pi} Q dh
\]

\[
\bar{Q}_{\text{day}} = \frac{S_0}{\pi} \frac{R_o^2}{R_E^2} [h_0 \sin \phi \sin \delta + \cos \phi \cos \delta \sin h_0]
\]

\(h_0\) denotes the hour angle when sun rises and can be calculated from

\[
\cos h_0 = -\tan \phi \tan \delta
\]

\[
h_0 = \begin{cases} 
180^\circ & \text{midnight (or always light)} \\
135^\circ & 3\text{am} \\
90^\circ & 6\text{am} \\
45^\circ & 9\text{am} \\
0^\circ & 12\text{pm (or always dark)}
\end{cases}
\]
Daily Average Insolation

Figure: Theoretical daily-average insolation at the top of the atmosphere.

\[ \bar{Q}_{\text{day}} \quad (\text{W} \ \text{m}^{-2}) \]

Although the average insolation over the whole planet is 341 W/m², the actual daily average varies significantly by season (particularly far from the equator).
Insolation and Climate

- Illustrated comparisons of insolation at different latitudes:
  - Equator: 1 unit
  - Mid Latitudes: 1.4 units, 1 unit
  - Poles: 2 units

*Images courtesy of IPAM, The COMET Program, Gifford Wong, PHI Inc., NSF.*
**Figure:** Annual mean insolation at (top) the top of Earth’s atmosphere and (bottom) at the planet’s surface.

Why is there a difference between the top and bottom figures?

A photon received from the sun has **five possible fates**:

- Scattering (in the atmosphere)
- Absorption (in the atmosphere)
- Reflection (in the atmosphere)
- Reflection (by the surface)
- Absorption (by the surface)
Scattering and Absorption

**Question:** What determines how much radiation is absorbed by the atmosphere?

**Answer:** There are different types of radiation, determined by the wavelength of each photon. The wavelength of the photon determines how it interacts with gases and particles in the atmosphere.
Electromagnetic Spectrum

- **Gamma Rays**: Gamma rays, X-rays and ultraviolet light blocked by the upper atmosphere (best observed from space).
- **X-Rays**: Visible light observable from Earth, with some atmospheric distortion.
- **Visible**: Most of the infrared spectrum absorbed by atmospheric gases (best observed from space).
- **Near IR**: Radio waves observable from Earth.
- **Far IR**: Long-wavelength radio waves blocked.

**Atmospheric Opacity**

- **Solar Radiation Peak Emission (~0.6μm)**
- **Terrestrial Radiation Peak Emission (~15μm)**
Scattering and Absorption

- Normalized intensity of radiation after passing through the atmosphere
- Total amount of absorption and scattering experienced by photons as they pass through the atmosphere
- Chemical constituents of the atmosphere responsible for absorption and scattering
Scattering and Absorption (Observations)

- The atmosphere is **almost completely transparent in the visible spectrum**.

- The atmosphere is **opaque in the ultraviolet spectrum**, in particular due to O$_2$ and O$_3$ absorption.

- The atmosphere has **variable opacity across the infrared spectrum**, being completely opaque at some wavelengths and transparent at others.

- The dominant constituent of the atmosphere, N$_2$ does not play into absorption at all. And O$_2$ is only important at the shortest wavelengths.

- Absorption of terrestrial radiation is dominated by triatomic molecules O$_3$, H$_2$O, N$_2$O and CO$_2$ plus CH$_4$. These molecules have rotational and vibrational modes and so can be excited by infrared radiation. They are present only in tiny concentrations but play a key role in the absorption of terrestrial radiation. They are examples of **greenhouse gases**.
**Reflection by the Surface and Atmosphere**

**Definition:** Albedo is the ratio of reflected solar energy to incident solar energy.

This figure depicts approximate ranges for the percentage of diffusively reflected sunlight (albedo) for a variety of surface types.

**Question:** Which of these surface types is most closely associated with a black body?
Reflection by the Atmosphere (Albedo)

Mean Annual Cloud Cover (Fraction)

Reflection by the Surface (Albedo)

Mapping terrestrial biomes around the world
https://en.wikipedia.org/wiki/Biome
Reflection by the Surface (Albedo)

https://visibleearth.nasa.gov/images/60636/global-albedo

Polar Albedo > 0.7
**Planetary Albedo**

**Definition:** Planetary Albedo $\alpha_p$ is the fraction of incoming solar radiation at the Earth reflected back to space averaged over the whole planet.

For the Earth, at present: $\alpha_p \approx 0.30$

Hence the solar radiation absorbed by the Earth is approximately given by

$$P_{abs} = (1 - \alpha_p) S_0 \pi a^2 = 1.22 \times 10^{17} \text{ W}$$
Energy Balance

Normalized to incoming solar = 100
Simple Models of Energy Balance
Since the Earth must satisfy energy balance, the total incoming radiation must be balanced with the outgoing radiation.

\[ P_{\text{abs}} = P_{\text{emit}} \]

\[ P_{\text{abs}} = (1 - \alpha_p) S_0 \pi a^2 \]

If we approximate the Earth as a black body, energy is emitted isotropically. Hence,

\[ P_{\text{emit}} = 4 \pi a^2 \sigma T_e^4 \]

**Definition:** The emission temperature of a planet is the black body temperature required to achieve energy balance. Denoted \( T_e \).

\[ T_e = \left[ \frac{S_0 (1 - \alpha_p)}{4 \sigma} \right]^{1/4} \]
Emission Temperature

\[ T_e = \left[ \frac{S_0(1 - \alpha_p)}{4\sigma} \right]^{1/4} \]

\[ \sigma = 5.670400 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4} \]

\[ \alpha_p = 0.3 \]

\[ S_0 \approx 1367 \text{ W m}^{-2} \]

Emission Temperature of the Earth:

\[ T_e = 255K \]
The calculated emission temperature of the Earth is 33 K cooler than the globally averaged observed surface temperature ($T_s$)

$$T_s = 288 \text{ K}$$

**Question:** Why might this be the case?
Greenhouse Effect

• Since the atmosphere is mostly opaque to IR radiation, it is not reasonable to assume that terrestrial radiation is directly radiated away into space.

• Instead an outgoing photon will often be absorbed and re-emitted several times, possibly even returning to the surface. This causes the surface to be much warmer than it would be if no atmosphere was present.

Definition: The greenhouse effect refers to the “trapping” of terrestrial radiation by greenhouse gasses in the atmosphere—namely, the blocking and re-emission of outgoing terrestrial radiation by the atmosphere.

Definition: Gases that contribute to the greenhouse effect by absorbing terrestrial radiation are referred to as greenhouse gases.
Assume the atmosphere consists of a single layer which is transparent to solar radiation, but opaque to infrared radiation.

Can we calculate the surface temperature in this scenario?
Greenhouse Effect (1 Layer Opaque)

Average net solar flux per unit area:

\[ F_0 = \frac{(1 - \alpha_p)S_0}{4} \]

Terrestrial radiation emitted to space:

\[ A^\uparrow = \sigma T_a^4 = \frac{1}{4}(1 - \alpha_p)S_0 \]

Downwelling flux emitted from atmosphere:

\[ A^\downarrow = \sigma T_a^4 = A^\uparrow \]

Surface energy balance:

\[ S^\uparrow = \sigma T_s^4 = \frac{1}{4}(1 - \alpha_p)S_0 + A^\downarrow \]

\[ \sigma T_s^4 = 2 \times \frac{1}{4}(1 - \alpha_p)S_0 \]

\[ T_s = 2^{1/4}T_e \]
Greenhouse Effect (1 Layer Opaque)

\[ T_s = 2^{1/4} T_e \]

So the presence of an opaque atmosphere increases the surface temperature (compared to the no-atmosphere case) by a factor of

\[ 2^{1/4} \approx 1.19 \]

Then using our calculated value of \( T_e \) we have

\[ T_s = \_\_\_ K \]

This approximation is much closer to our expected value, but is now an overestimate!

Why might this be the case?
Not all solar flux incident on the top of the atmosphere reaches the surface (the atmosphere is not completely opaque to solar radiation).

A completely opaque atmosphere is too extreme – as seen earlier, some IR emissions are still allowed through.
Assume the atmosphere consists of a single layer which is transparent to solar radiation, but partially opaque to infrared radiation.

Can we calculate the surface temperature in this scenario?
Greenhouse Effect (1 Layer Leaky)

Average net solar flux per unit area:

\[ F_0 = \frac{(1 - \alpha_p)S_0}{4} \]

Terrestrial radiation emitted to space:

\[ F^\uparrow = (1 - \epsilon)S^\uparrow + A^\uparrow \]

Energy balance:

\[ F_0 = F^\uparrow \]

Atmospheric equilibrium:

\[ A^\uparrow = A^\downarrow \]

Zero net flux at surface:

\[ \frac{1}{4}(1 - \alpha_p)S_0 + A^\downarrow = S^\uparrow \]

\[ S^\uparrow = \sigma T_s^4 = \frac{1}{2(2 - \epsilon)}(1 - \alpha_p)S_0 \]

\[ = \frac{2}{2 - \epsilon} \sigma T_e^4 \]

\[ T_s = \left( \frac{2}{2 - \epsilon} \right)^{1/4} T_e \]
Greenhouse Effect (1 Layer Leaky)

\[ T_s = \left( \frac{2}{2 - \epsilon} \right)^{1/4} T_e \]

Consider limits \( \epsilon \to 0 \) and \( \epsilon \to 1 \)

How does this atmospheric model compare?
The assumption that space and the surface “see” the same atmosphere is also an approximation.

The conceptual model can then be modified to include a second layer. To find surface temperature, repeat the energy balance procedure.
Of course, two atmospheric layers is again an approximation. To obtain a “real” result, the atmosphere must be divided into an infinite number of layers with absorption coefficients dependent on concentrations of H₂O, CO₂ and O₃.
The resulting temperature profile would be the actual mean atmospheric temperature profile if the exchange of energy in the atmosphere was only through radiative transfer.

This model has a temperature discontinuity at the surface which is not observed in reality. In fact, such a discontinuity would immediately trigger atmospheric convection in the real atmosphere.

Figure 2.11 from Marshall and Plumb (adapted from Wells 1997)
Climate Sensitivity

- The greenhouse models illustrate important radiative feedbacks that play a critical role in regulating the climate of the planet.

- Recall that the two-layer leaky atmosphere model had two parameters that we were free to choose: $S_0$ and $\varepsilon$.

- Observe that changing these parameters will also change the equilibrium temperature of the surface and atmosphere.

- **Definition:** Climate sensitivity is the change in surface temperature induced by a change in some parameter $Q$.

  Represented mathematically as $\frac{\partial T_s}{\partial Q} \Delta Q$.
In the climate literature, climate sensitivity usually refers to carbon dioxide sensitivity. This quantity represents the change in surface temperature which would be caused by a doubling in atmospheric carbon dioxide.

- This value is currently estimated as $3 \pm 1.5 ^\circ C$

- **Question:** What is the climate sensitivity of the two-layer model with $\varepsilon = 0$ to changes in $S_0$?
• Complicating the measurement of climate sensitivity is the presence of climate feedbacks (more on this later).

• A positive climate feedback is one which enhances the initial change.

• A negative climate feedback is one which reduces the initial change.

**Question:** Can you identify the climate feedbacks in this image?
Climate Feedbacks

The Bretherton diagram (NASA 1986) is a famous visualization of the interconnectedness of the Earth system.
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Thank You!