Part 5: A Developing Surface Low
Vertical Motion

Continuity Equation

\[
\left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_p + \frac{\partial \omega}{\partial p} = 0
\]

Integrate both sides

\[
\int_{\omega(p=p')}^{\omega(p=0)} d\omega = - \int_{p=p'}^{p=0} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_p dp
\]

This equation formally links vertical pressure velocity and horizontal divergence on pressure surfaces.
Vertical Motion

Recall the equation for $\omega$:

$$\omega = \frac{\partial p}{\partial t} + \left( u_a \frac{\partial p}{\partial x} + v_a \frac{\partial p}{\partial y} \right) - w g \rho$$

Local change in pressure:

$$\frac{\partial p}{\partial t} \approx \frac{U \Delta P}{L} \approx 10^{-2} \text{ Pa s}^{-1}$$

Pressure advection by ageostrophic wind:

$$u_a \cdot \nabla_h \rho \approx 0.1 \times \frac{U \Delta P}{L} \approx 10^{-3} \text{ Pa s}^{-1}$$

Vertical velocity term:

$$w g \rho \approx W g \rho \approx 10^{-1} \text{ Pa s}^{-1}$$

$\omega = 0$ at the surface
Vertical Motion

At the surface we instead have \[ \omega \approx \frac{\partial p}{\partial t} \]

\[ \omega(p') = - \int_0^{p'} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_p \, dp \]

\[ \omega(p_s) \approx - \int_0^{p_s} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_p \, dp \]

\( p_s \) denotes surface pressure

Convergence of mass into the column above will increase the surface pressure. Divergence of mass will decrease the surface pressure.
Development of a Surface Low

Earth's surface

pressure surfaces
Development of a Surface Low

Warming

Earth’s surface

pressure surfaces
Development of a Surface Low

Warming increases thickness

pressure surfaces

Earth’s surface
Development of a Surface Low

Earth’s surface

mass diverges up here

warming increases thickness

Warming
Development of a Surface Low

Warming increases thickness
mass diverges up here

generates low here
Earth’s surface
Development of a Surface Low

mass diverges up here

generates low here

and highs here

Earth’s surface

Warming

HIGH

LOW

HIGH

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Development of a Surface Low

LOW pressure gradient initiates convergence down here

mass diverges up here

Warming

Earth’s surface

HIGH LOW HIGH

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