Applications of the Basic Equations
Chapter 3

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Part 3: The Thermal Wind
**Question:** Is there a relationship between wind speed and temperature?

It turns out that there is a close link between \textit{vertical wind shear} (vertical gradients of horizontal wind speed) and \textit{layer thickness}, which is governed by temperature.
Question: Where are the strongest temperature gradients?
Wind speeds appear to be largest where temperature gradients in lower layers are strongest.
**Thermal Wind**

**Definition:** The thermal wind is a vector difference between the geostrophic wind at an upper level and a lower level.

**Figure:** Thickness of layers related to temperature, causes a tilt in the pressure surfaces.

Change in magnitude of horizontal gradient of pressure then leads to vertical wind shear.
Thermal Wind

**Emphasis:** The thermal wind is not a real wind, but a vector difference.

The thermal wind vector points such that **cold air is to the left** and **warm air is to the right**, parallel to isotherms (in the northern hemisphere). Cold air is to the right and warm air is to the left in the southern hemisphere.
**Geostrophic Wind**

*In Pressure Coordinates*

**Recall:** The **geostrophic wind** is the component of the real wind which is governed by geostrophic balance. On constant height surfaces, it is defined to satisfy

\[
\begin{align*}
    u_g &= -\frac{1}{f \rho} \frac{\partial p}{\partial y} \\
    v_g &= +\frac{1}{f \rho} \frac{\partial p}{\partial x}
\end{align*}
\]

In pressure coordinates, this relationship takes on the analogous form:

\[
\begin{align*}
    u_g &= -\frac{1}{f \rho} \frac{\partial \Phi}{\partial y} \\
    v_g &= +\frac{1}{f \rho} \frac{\partial \Phi}{\partial x}
\end{align*}
\]
Geostrophic Wind

\[ u_g = -\frac{1}{f} \left( \frac{\partial \Phi}{\partial y} \right)_p \]
\[ v_g = \frac{1}{f} \left( \frac{\partial \Phi}{\partial x} \right)_p \]

Differentiate with respect to \( p \)

\[ \frac{\partial u_g}{\partial p} = -\frac{1}{f} \frac{\partial}{\partial y} \frac{\partial \Phi}{\partial p} \]
\[ \frac{\partial v_g}{\partial p} = \frac{1}{f} \frac{\partial}{\partial x} \frac{\partial \Phi}{\partial p} \]

Hydrostatic Relationship

\[ \frac{\partial \Phi}{\partial p} = g \frac{\partial z}{\partial p} = -\frac{1}{\rho} = -\frac{RT}{p} \]

On constant \( p \) surfaces

Thermal wind relationship:

\[ \frac{\partial u_g}{\partial p} = \frac{R}{pf} \left( \frac{\partial T}{\partial y} \right)_p \]
\[ \frac{\partial v_g}{\partial p} = -\frac{R}{pf} \left( \frac{\partial T}{\partial x} \right)_p \]
Thermal wind relationship:

\[
\frac{\partial u_g}{\partial p} = \frac{R}{p f} \left( \frac{\partial T}{\partial y} \right)_p \quad \frac{\partial v_g}{\partial p} = -\frac{R}{p f} \left( \frac{\partial T}{\partial x} \right)_p
\]

This relationship links the horizontal temperature gradient with the vertical wind gradient (vertical shear).
Thermal Wind

Thermal wind relationship:
\[
\frac{\partial u_g}{\partial p} = \frac{R}{pf} \left( \frac{\partial T}{\partial y} \right)_p \\
\frac{\partial v_g}{\partial p} = -\frac{R}{pf} \left( \frac{\partial T}{\partial x} \right)_p
\]

The thermal wind itself is a vector difference.

Rewrite
\[
\frac{\partial u_g}{\partial (\log p)} = \frac{R}{f} \left( \frac{\partial T}{\partial y} \right)_p \\
\frac{\partial v_g}{\partial (\log p)} = -\frac{R}{f} \left( \frac{\partial T}{\partial x} \right)_p
\]

Integrate
\[
\begin{align*}
U_T &= u_g(p_2) - u_g(p_1) = -\frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial y} \right)_p \log \left( \frac{p_1}{p_2} \right) \\
V_T &= v_g(p_2) - v_g(p_1) = \frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial x} \right)_p \log \left( \frac{p_1}{p_2} \right)
\end{align*}
\]
Thermal Wind

\[ u_T = u_g(p_2) - u_g(p_1) = -\frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial y} \right)_p \log \left( \frac{p_1}{p_2} \right) \]

\[ v_T = v_g(p_2) - v_g(p_1) = \frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial x} \right)_p \log \left( \frac{p_1}{p_2} \right) \]

**Example:** Thermal wind \( v_T \) between 500 hPa and 1000 hPa

\[ u_g(p_2 = 500 \text{ hPa}) \]

\[ u_T \] thermal wind

\[ u_g(p_1 = 1000 \text{ hPa}) \]
Thermal Wind

Alternate form of thermal wind, written in terms of geopotential height (obtained from hypsometric equation):

\[ \mathbf{u}_T = \frac{R_d}{f} \mathbf{k} \times \nabla_p \langle T \rangle \log \left( \frac{p_1}{p_2} \right) = \frac{1}{f} \mathbf{k} \times \nabla_p (\Phi_2 - \Phi_1) \]

Alternate form of thermal wind, written in terms of geopotential height (obtained from hypsometric equation):

\[ u_T = -\frac{1}{f} \frac{\partial}{\partial y} (\Phi_2 - \Phi_1) \quad v_T = \frac{1}{f} \frac{\partial}{\partial y} (\Phi_2 - \Phi_1) \]

Index “1” indicates lower level, “2” indicates upper level.
Note that thermal wind always points parallel to lines of constant layer temperature:

\[ u_T \approx u_g(p_2) - u_g(p_1) = -\frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial y} \right)_p \log \left( \frac{p_1}{p_2} \right) \]

\[ v_T \approx v_g(p_2) - v_g(p_1) = \frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial x} \right)_p \log \left( \frac{p_1}{p_2} \right) \]

\[ \mathbf{u}_T \cdot \nabla \langle T \rangle = \frac{R}{f} \log \left( \frac{p_1}{p_2} \right) \left[ -\frac{\partial \langle T \rangle}{\partial y} \frac{\partial \langle T \rangle}{\partial x} + \frac{\partial \langle T \rangle}{\partial x} \frac{\partial \langle T \rangle}{\partial y} \right] = 0 \]
**Thermal Wind**

Thermal wind always points parallel to lines of constant temperature (and lines of constant layer thickness).

**Definition: Veering winds** are defined by clockwise rotation of the geostrophic wind with height and are associated with *warm air advection*.

**Definition: Backing winds** are defined by counter-clockwise rotation of the geostrophic wind with height and are associated with *cold air advection*.
The thermal wind determines the relationship between meridional temperature gradients and zonal winds.

**Question:** Given zonal mean temperature below, where are zonal jets?
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Question: Given zonal mean temperature below, where are zonal jets?

\[
\frac{\partial u_g}{\partial p} = \frac{R}{\rho f} \left( \frac{\partial T}{\partial y} \right)_p \frac{\partial u_g}{\partial z} \sim -\frac{\partial u_g}{\partial p}
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Question: Given zonal mean temperature below, where are zonal jets?

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