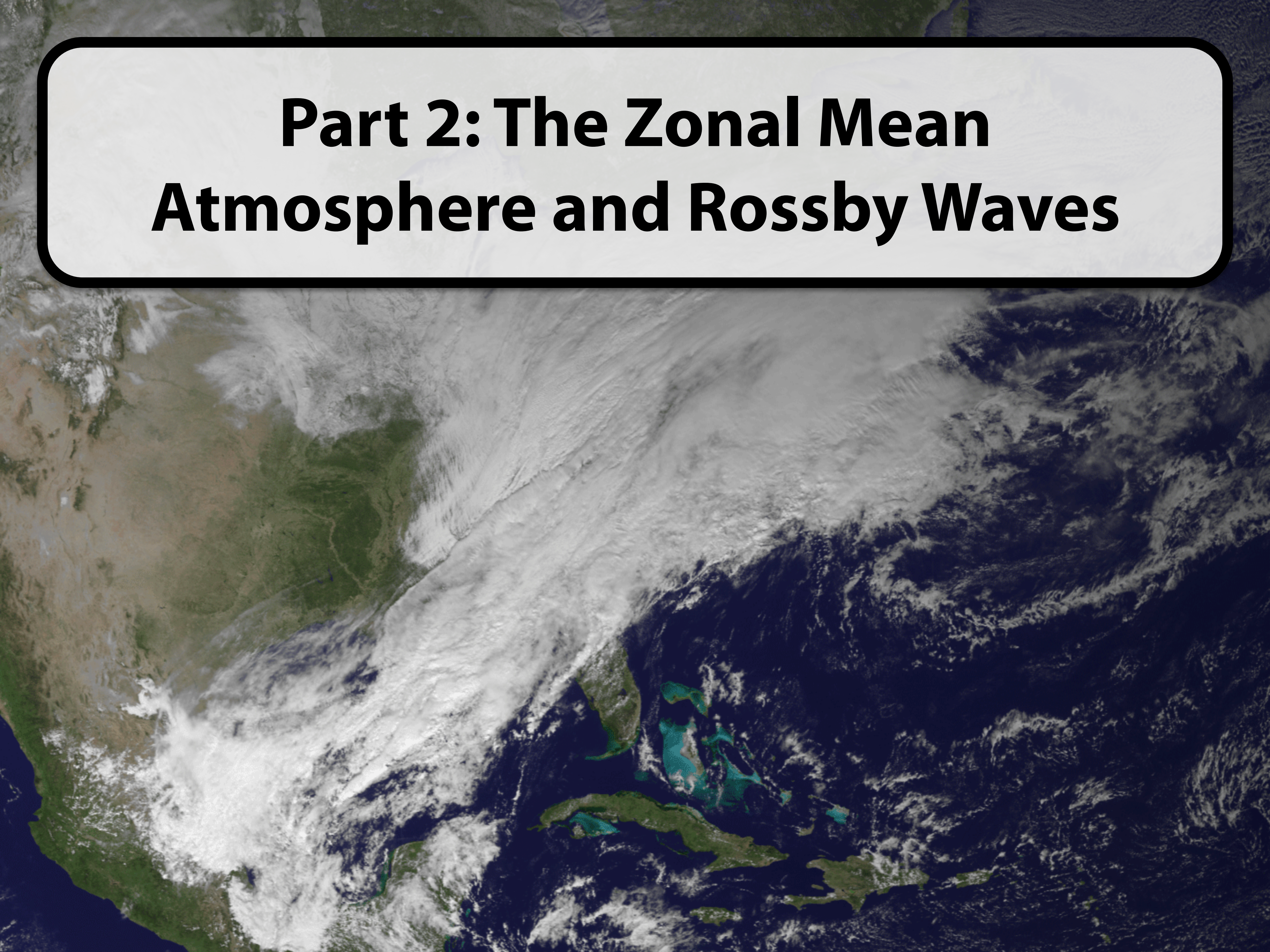
Atmospheric Waves
Chapter 6

Paul A. Ullrich
paullrich@ucdavis.edu

Part 2: The Zonal Mean Atmosphere and Rossby Waves



The Global Mean State

$$u = \overline{u} + u'$$

Variable

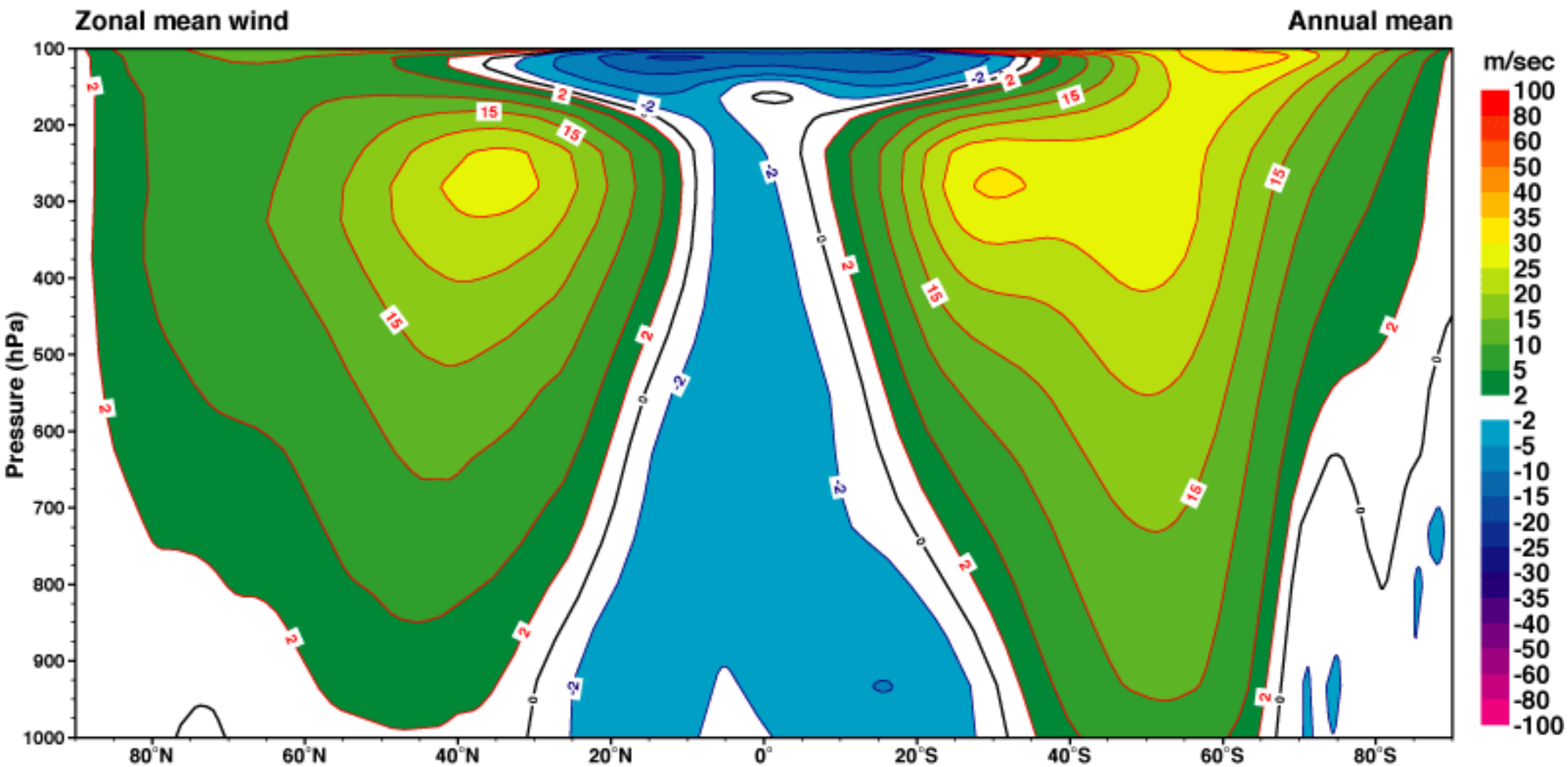
Mean

Difference (perturbation)

A mean state needs to be known **for each variable** that is involved in linearization to ensure that differences are actually small.

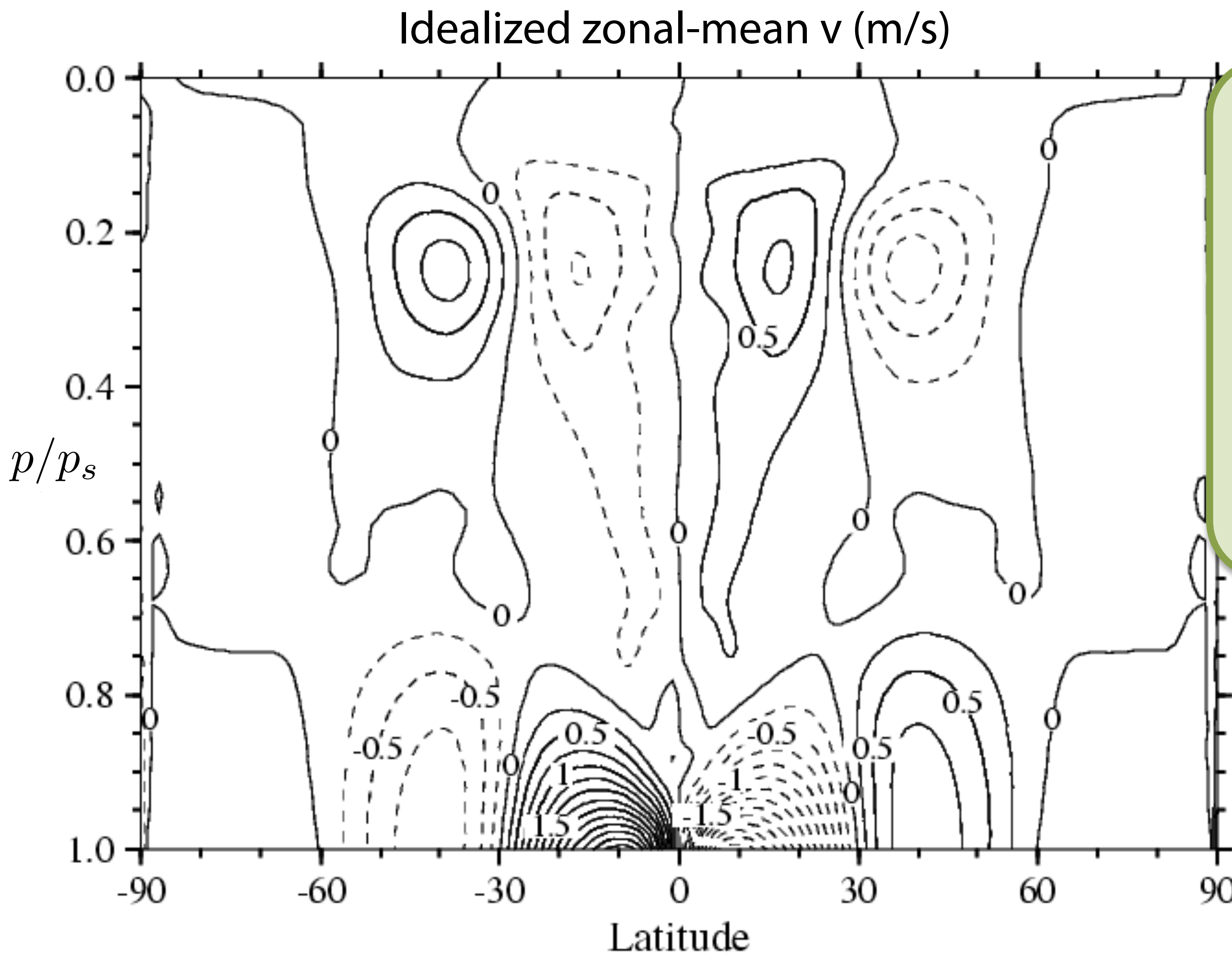
Question: What are appropriate choices for the mean state of the atmosphere?

Zonal Mean Zonal Wind



Constant basic-state zonal wind $\overline{u} \neq 0$ is a good assumption when analyzing linear waves, e.g. in midlatitudes. Also, $\overline{u}$ somewhat dependent on height.

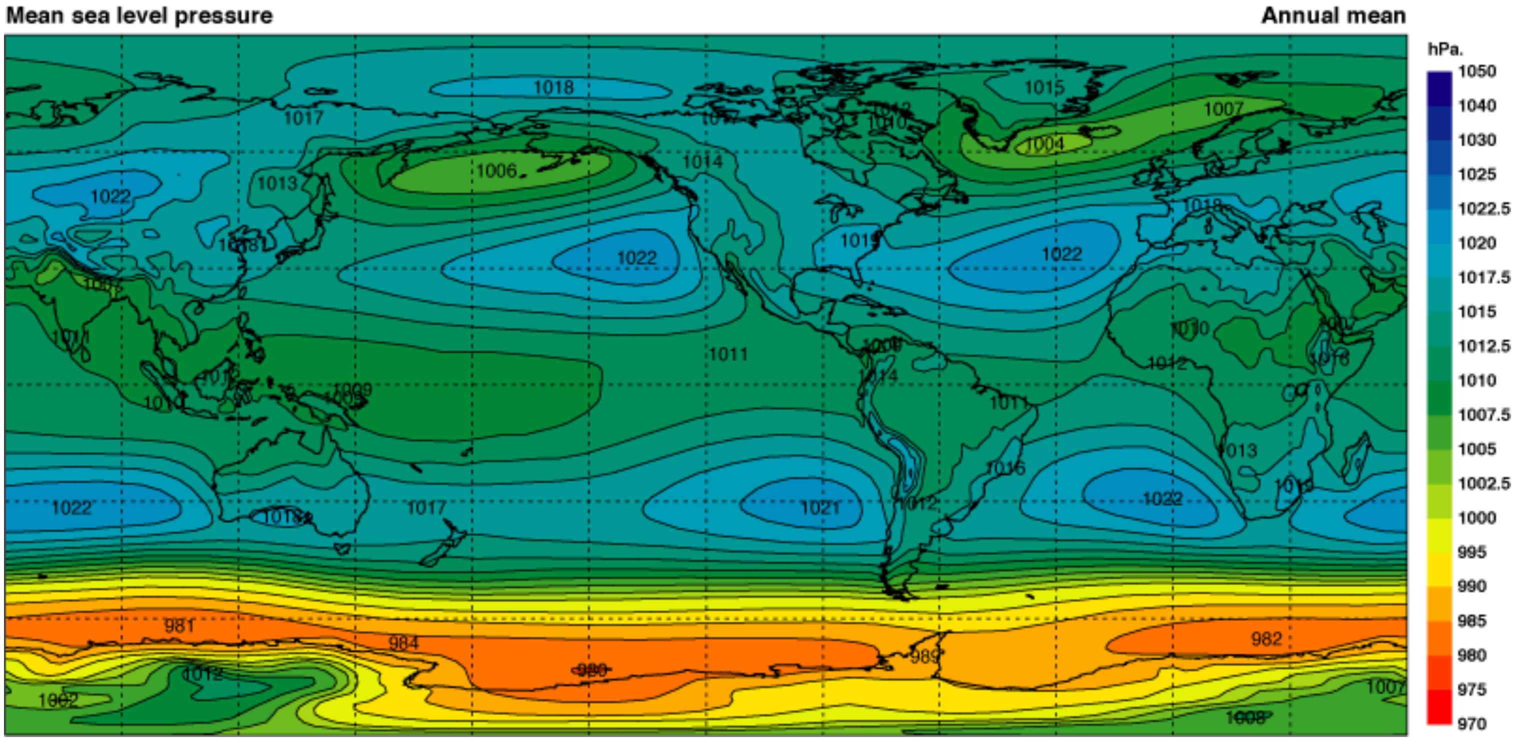
Zonal Mean Meridional Wind



Zonal-mean v is very small, therefore good assumption to use a basic state for linear waves.

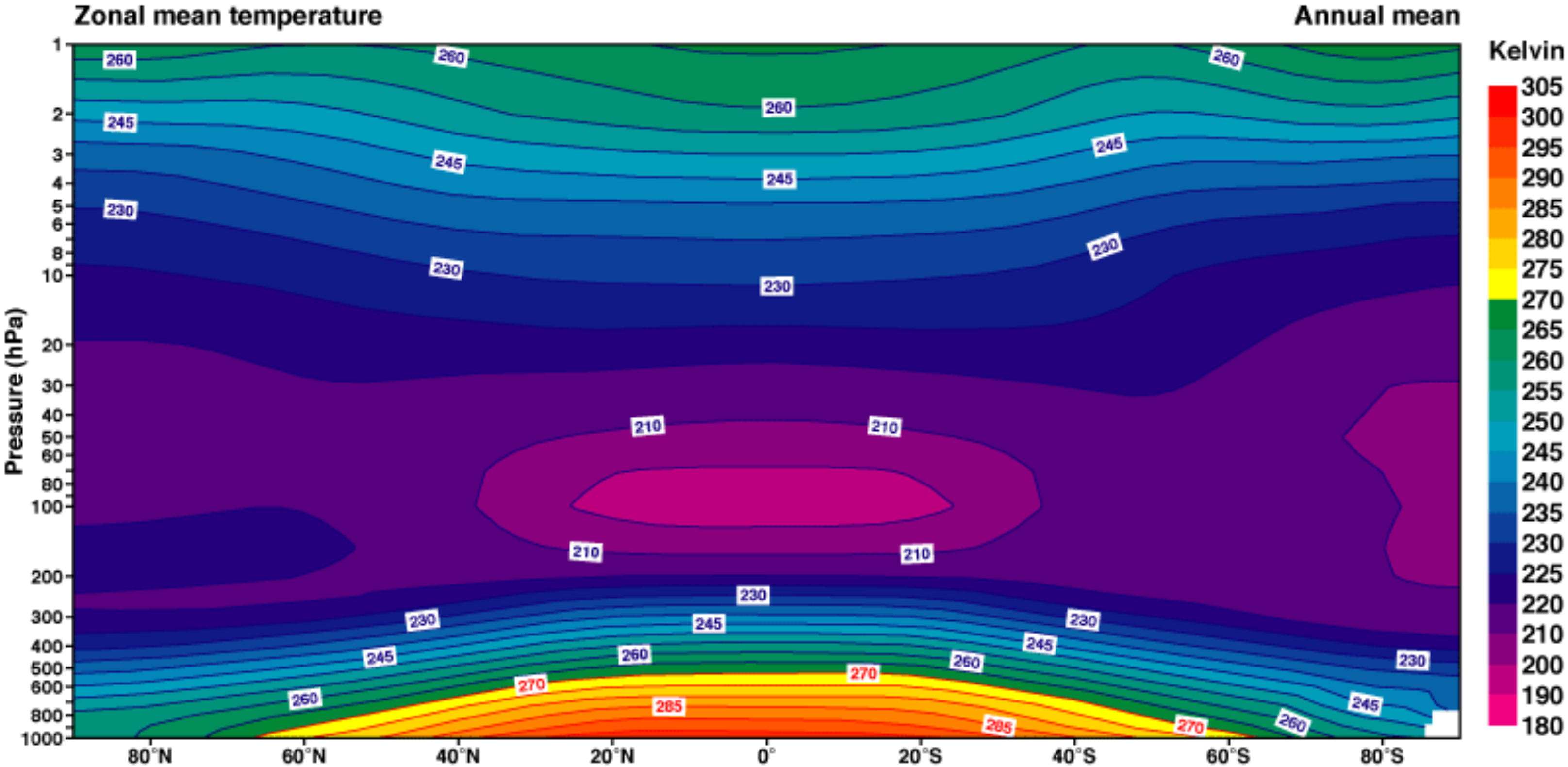
$$\overline{v} = 0$$

Annual Mean Zonal Mean Sea Level Pressure (MSLP)



Constant basic-state horizontal pressure is a good assumption, but *strong* pressure variations in the vertical direction: select height-dependent $\overline{p}(z)$

Annual Mean Zonal Mean Temperature



Constant basic-state horizontal temperature field is an okay assumption, but *strong* temperature variations in the vertical direction: select height-dependent $\overline{T}(z)$

US Standard Atmosphere

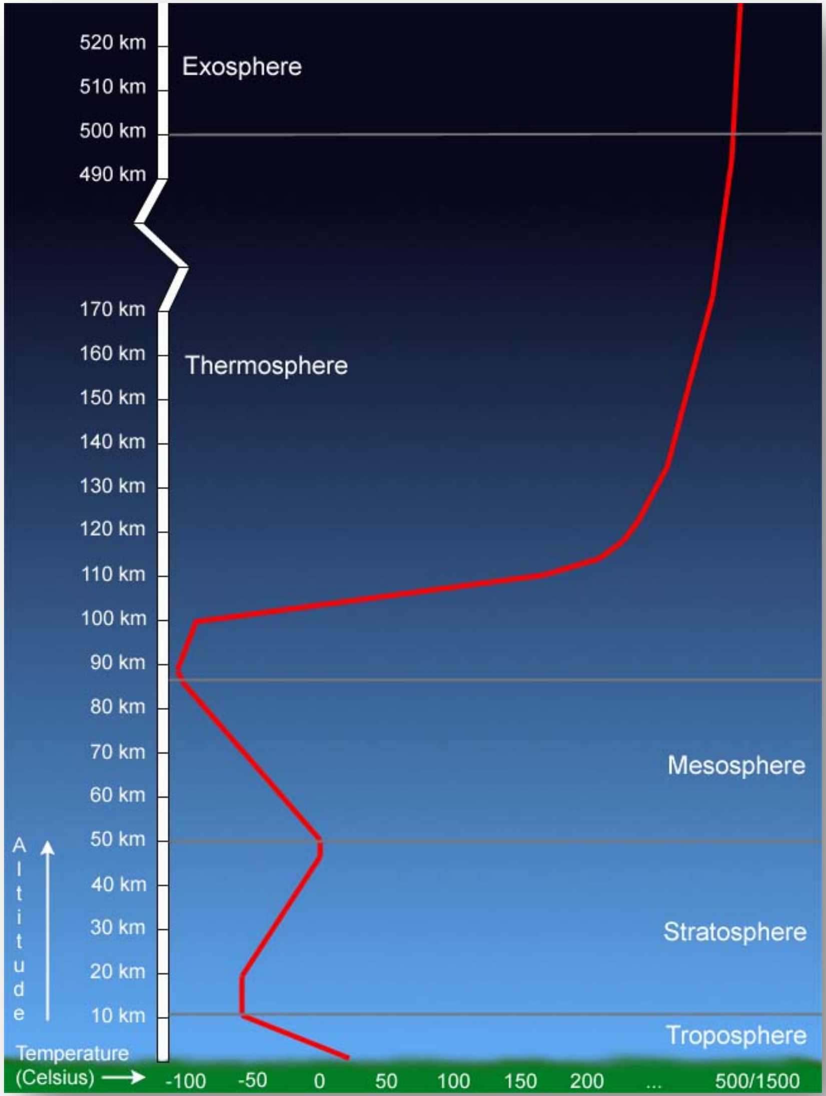
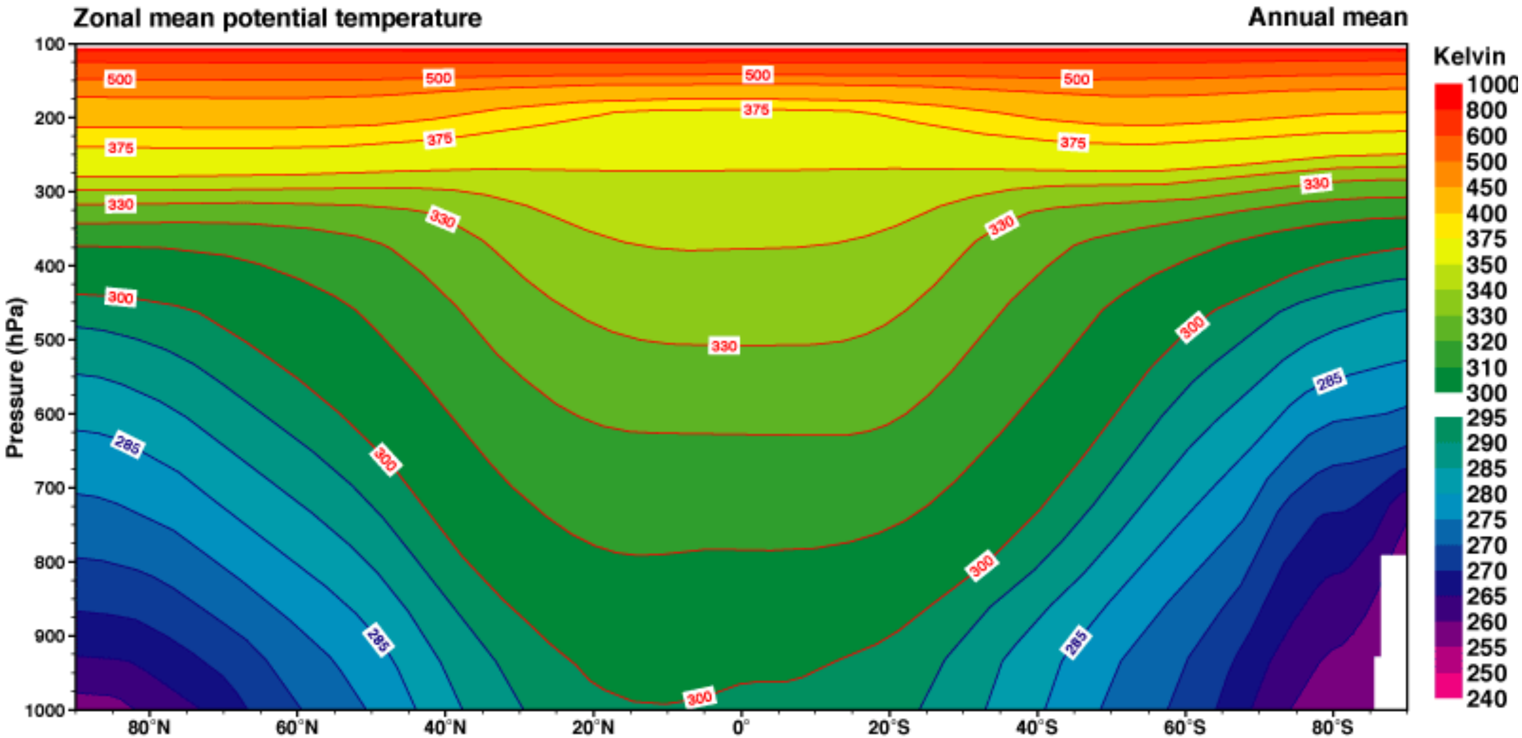


Figure: Typical temperature profile with height: U.S. standard atmosphere $\overline{T}(z)$

Annual Mean Zonal Mean Potential Temperature



Constant basic-state horizontal potential temperature field is an okay assumption, especially at mid- and high levels, but *strong* θ variations in the vertical: Select $\overline{\theta}(z)$

Rossby Waves

Also called **planetary waves**. These are the most important wave for large-scale atmospheric processes.

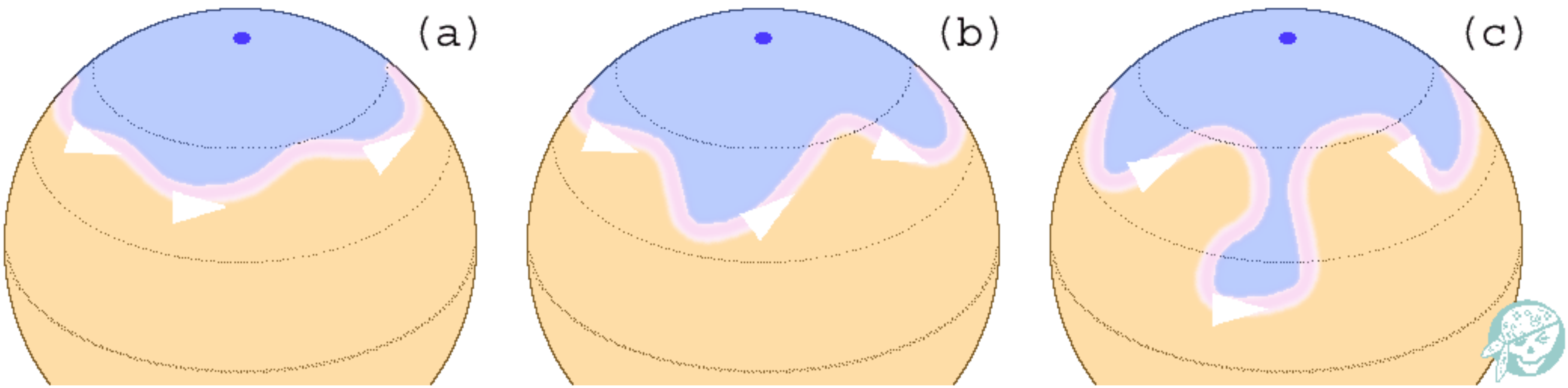
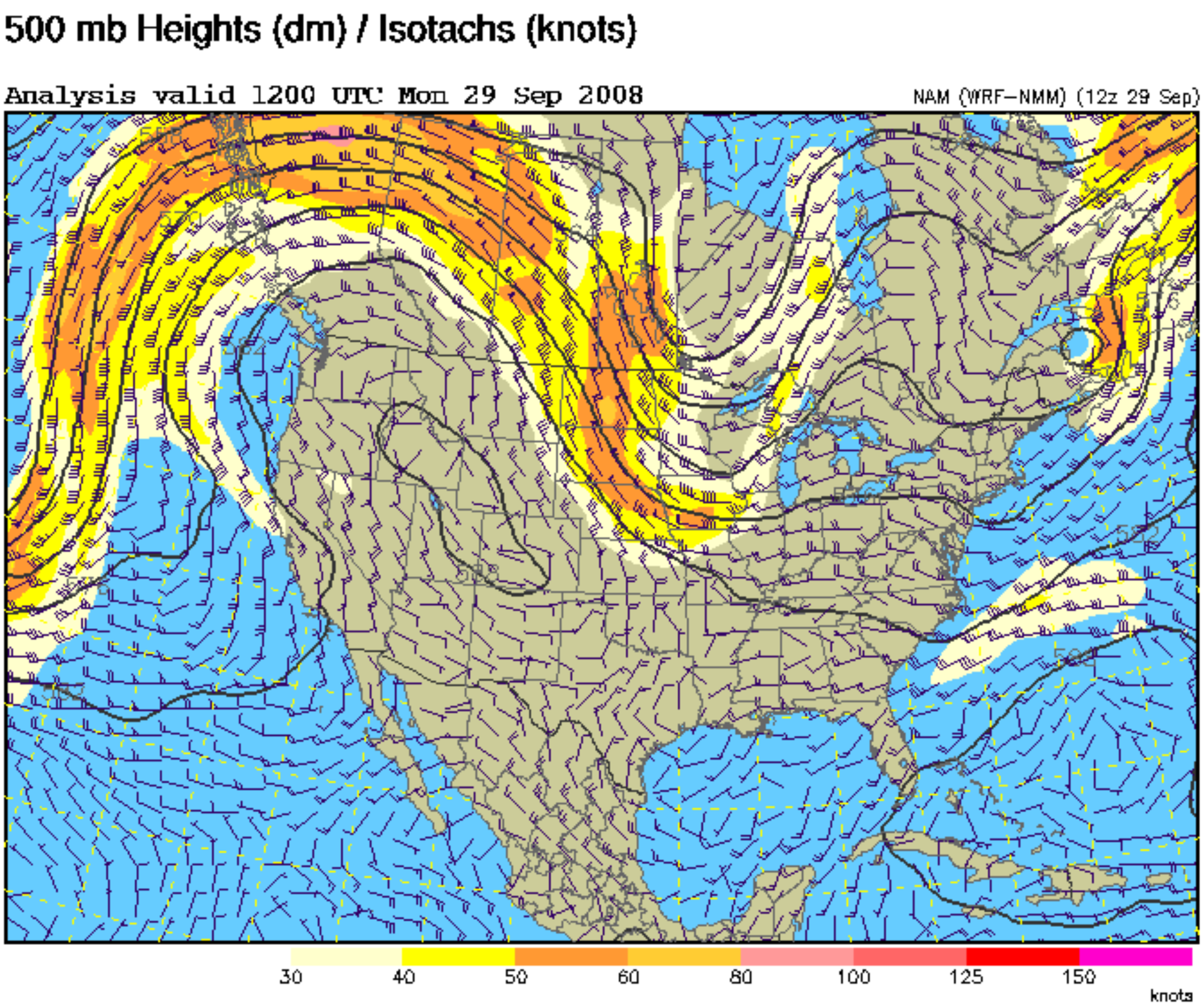


Figure: Large-scale meanders (usually 4-6) of the jet stream develop (a, b) and finally detach a "drop" of cold air (c). [orange: warmer masses of air; pink: jet stream]

Synoptic-Scale Rossby Waves



Source: http://www.rap.ucar.edu/weather/model/index.php?model=eta

Barotropic Rossby Waves

Rossby wave can be studied (conceptionally) in an inviscid (no friction) barotropic fluid of constant depth.

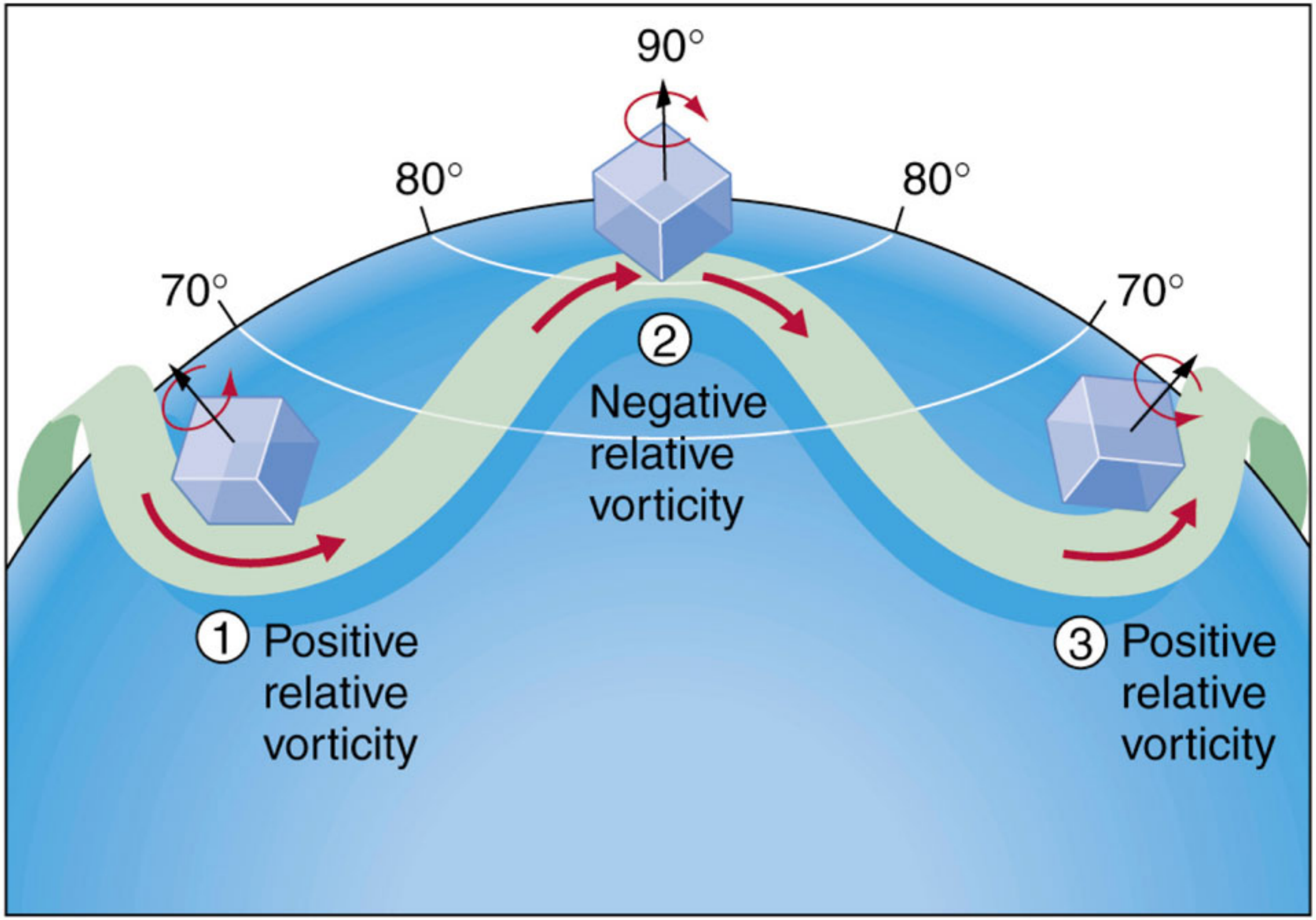
Such a system conserves the absolute vorticity:

$$\frac{D_h}{Dt}(\zeta + f) = 0 \quad \Longrightarrow \quad \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} \right) \zeta + \beta v = 0$$

Barotropic Rossby waves owe their existence to the variation of the Coriolis parameter with latitude, the so-called **β-effect**.

Rossby waves can be understood in a qualitative fashion by considering a closed chain of fluid parcels initially aligned along a circle of latitude.

Barotropic Rossby Waves



Barotropic Rossby Waves

Starting from: Conservation of absolute vorticity

$$\frac{D_h \eta}{Dt} = 0 \quad \eta = \zeta + f$$

Assume that $\zeta = 0$ at time t_1

Suppose that at time t_1, δy is the meridional displacement of a fluid parcel from the original latitude:

$$\zeta(t_1) + f(t_1) = f(t_0)$$

$$\zeta(t_1) = f(t_0) - f(t_1) = -\beta \delta y$$

Chain of parcels is subject to a sinusoidal meridional displacement, conserves η :

Positive relative vorticity for southward displacement

Negative relative vorticity for northward displacement

Barotropic Rossby Waves

Compute **phase speed *c*** of the propagating wave where a is the maximum northward displacement:

$$\delta y = a \sin\left[k(x - ct)\right]$$

$$v = \frac{D}{Dt}(\delta y) = \left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)(\delta y) = ka(\overline{u} - c)\cos\left[k(x - ct)\right]$$

$$\zeta = \frac{\partial v}{\partial x} = -k^2 a(\overline{u} - c)\sin\left[k(x - ct)\right]$$

$$\zeta(t_1) = -\beta \delta y$$

$$-k^2 a(\overline{u} - c)\sin\left[k(x - ct)\right] = -\beta a \sin\left[k(x - ct)\right]$$

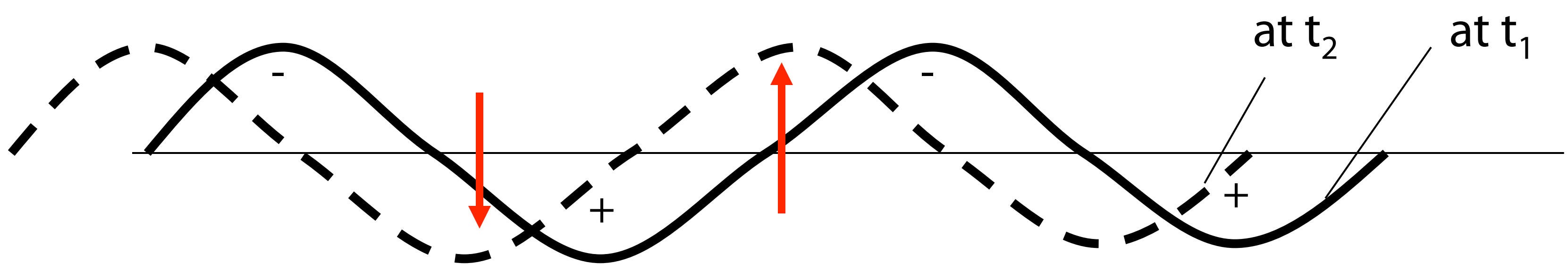
$$c = \overline{u} - \frac{\beta}{k^2}$$

Observe: Rossby waves always propagate westward relative to the mean flow.

Barotropic Rossby Waves

The perturbation vorticity field induces a meridional velocity field which advects parcels:

- Southward: West of vorticity maximum
- Northward: West of vorticity minimum



- Parcels will oscillate about their equilibrium latitude
- Pattern of vorticity maxima and minima **propagates to the west** (characteristic of Rossby waves)
- Restoring mechanism for Rossby waves: **Meridional gradient of absolute vorticity** (resists meridional displacements)

Barotropic Rossby Waves

Alternative derivation

Start with barotropic vorticity equation on a midlatitude β-plane:

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} \right) \zeta + \beta v = 0$$

Linearize around a constant basic state zonal velocity plus a small perturbation:

$$u = \overline{u} + u' \qquad v = v' \qquad \zeta = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} = \zeta'$$

Define a perturbation streamfunction Ψ' according to

$$u' = -\frac{\partial \Psi'}{\partial y} \qquad y' = \frac{\partial \Psi'}{\partial x} \qquad \zeta' = \nabla^2 \Psi'$$

Barotropic Rossby Waves

Alternative derivation

Leads to: Perturbation form of the barotropic vorticity equation

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)\nabla^2\Psi' + \beta\frac{\partial\Psi'}{\partial x} = 0$$

Seek solutions of the form

$$\Psi' = \Psi \exp\left[i(kx + \ell y - \nu t)\right]$$

And (at the end) pick the real part of the solution.

Barotropic Rossby Waves

Dispersion relation: $$\nu = \overline{u}k - \frac{\beta k}{\kappa^2}$$

Phase speed: $$c_x = \overline{u} - \frac{\beta}{\kappa^2} \qquad c_y = \frac{k\overline{u}}{\ell} - \frac{k\beta}{\ell\kappa^2} \qquad \boxed{\kappa^2 = k^2 + \ell^2}$$

Group speed: $$c_{g,x} = \overline{u} + \frac{\beta(k^2 - \ell^2)}{\kappa^4} \qquad c_{g,y} = -\frac{2\beta k\ell}{\kappa^4}$$

Rossby wave phase speed **westward** relative to the mean flow

Phase speed depends **inversely** on the **square of the horizontal wavenumber**

Therefore: Rossby waves are **dispersive**, phase speeds increase with increasing wavelength.

Barotropic Rossby Waves

Alternative derivation

Consistent with the discussion from quasi-geostrophic theory:

> *If the advection of planetary vorticity dominates over the advection of relative vorticity (e.g. for long wavelength), waves move westward (retrogress).*

Example: For $k \approx \ell$ and zonal wavelength of order 6000 km, Rossby wave speed is $\approx$ *-8 m/s* relative to mean flow

- (Long) Rossby waves may become stationary ($c=0$) relative to the surface of the Earth if the wavenumber equals the stationary wavenumber.

Stationary Wavenumber

$$\kappa^2 = \frac{\beta}{\overline{u}} = \kappa_s^2$$

Barotropic Rossby Waves

- Unlike the phase speed (always westward), the zonal group velocity may be either westward or eastward relative to the main flow.

- The direction depends on the ratio of the zonal and meridional wavenumbers.

- Stationary and most synoptic-scale Rossby waves have eastward zonal group velocities.

- Synoptic waves: **eastward advection by mean *u* often dominates** over westward phase propagation of Rossby waves, waves most eastward but at slower phase speed than zonal group velocity.

- Important consequence: new disturbances tend to develop **downstream** of existing disturbances.

Synoptic-Scale Rossby Waves

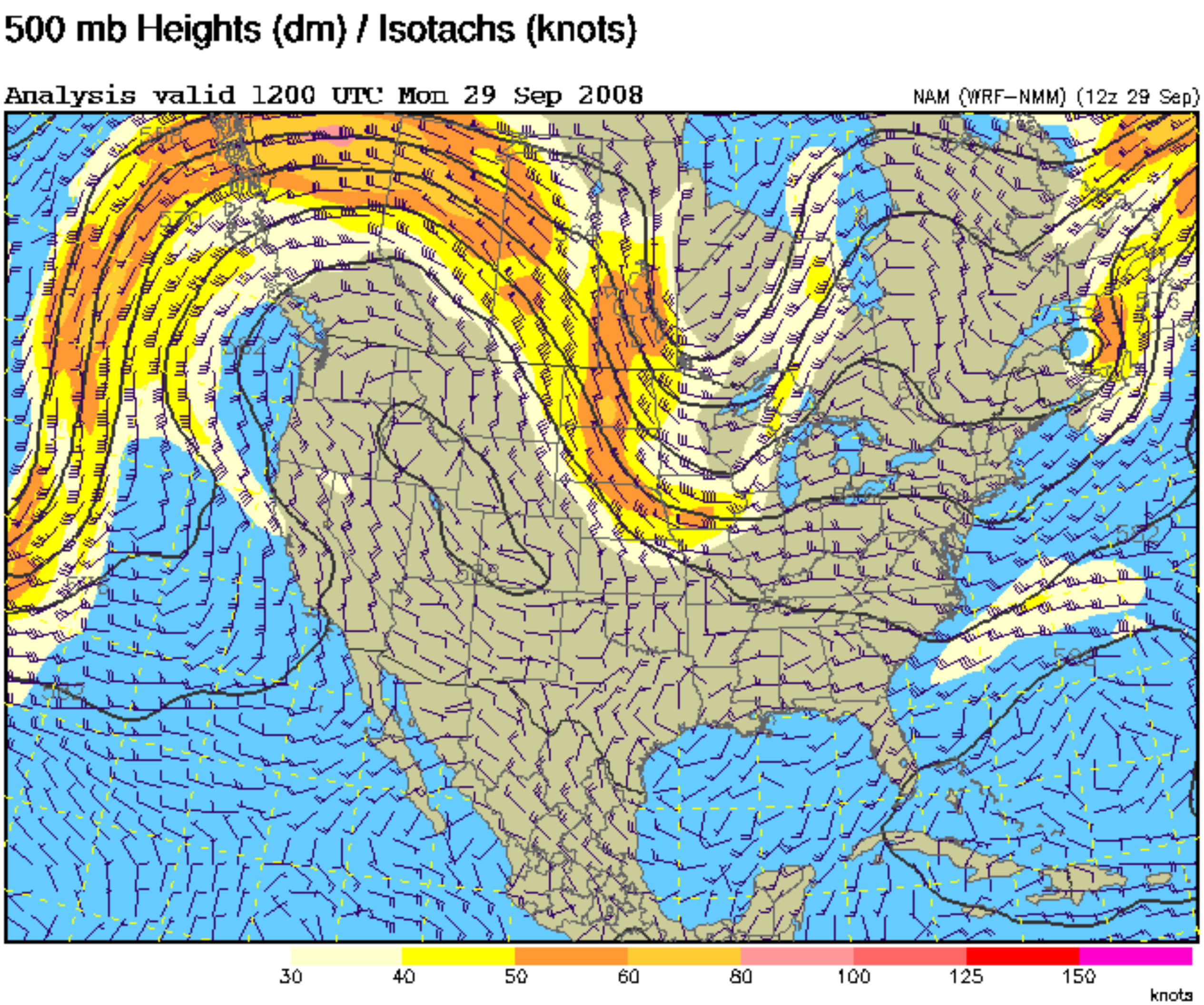


Figure: Midlatitude weather systems travel eastward, as advection by mean flow dominates.

Source: http://www.rap.ucar.edu/weather/model/index.php?model=eta