Applications of the Basic Equations

Chapter 3

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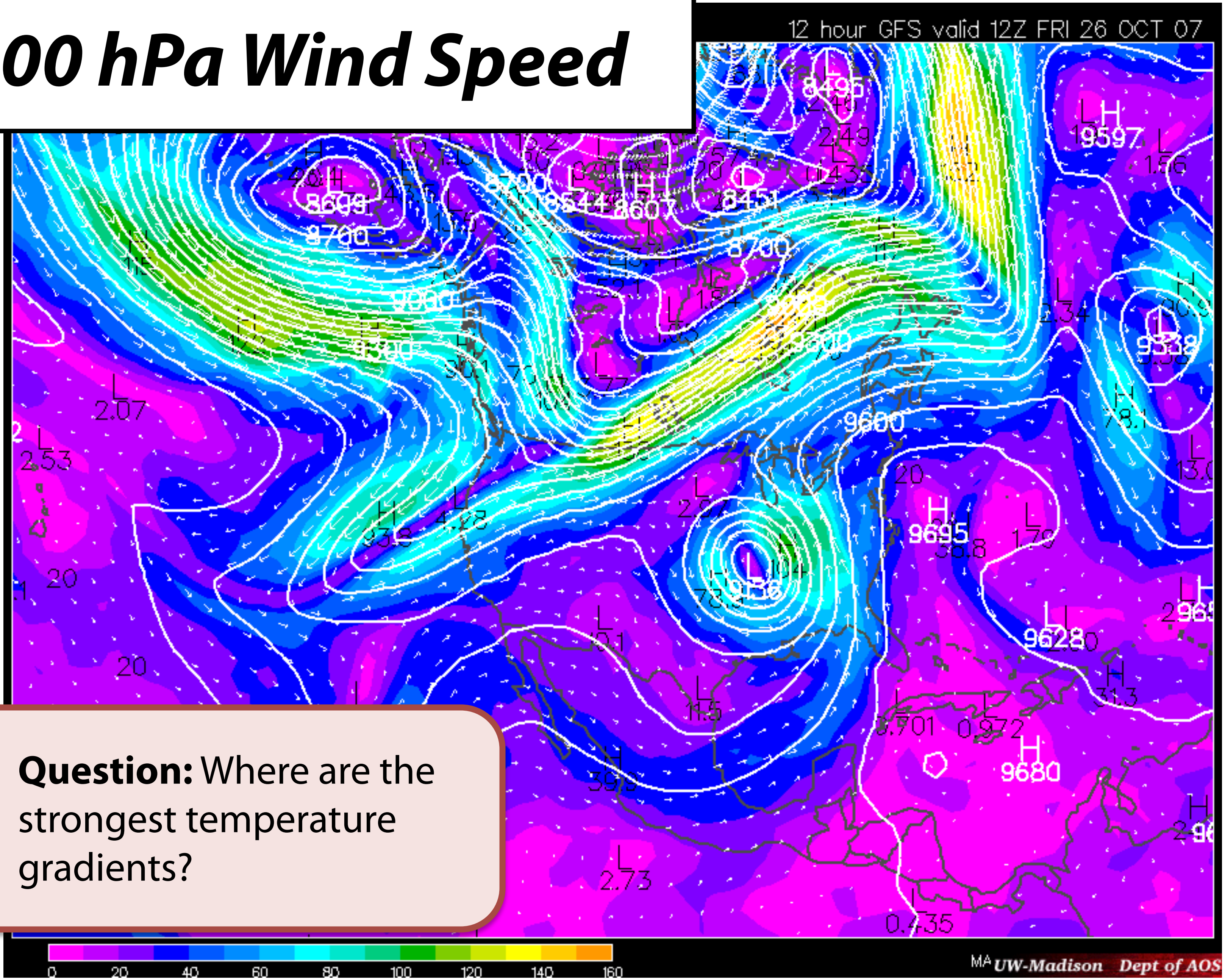
Part 3: The Thermal Wind

Question: Is there a relationship between wind speed and temperature?

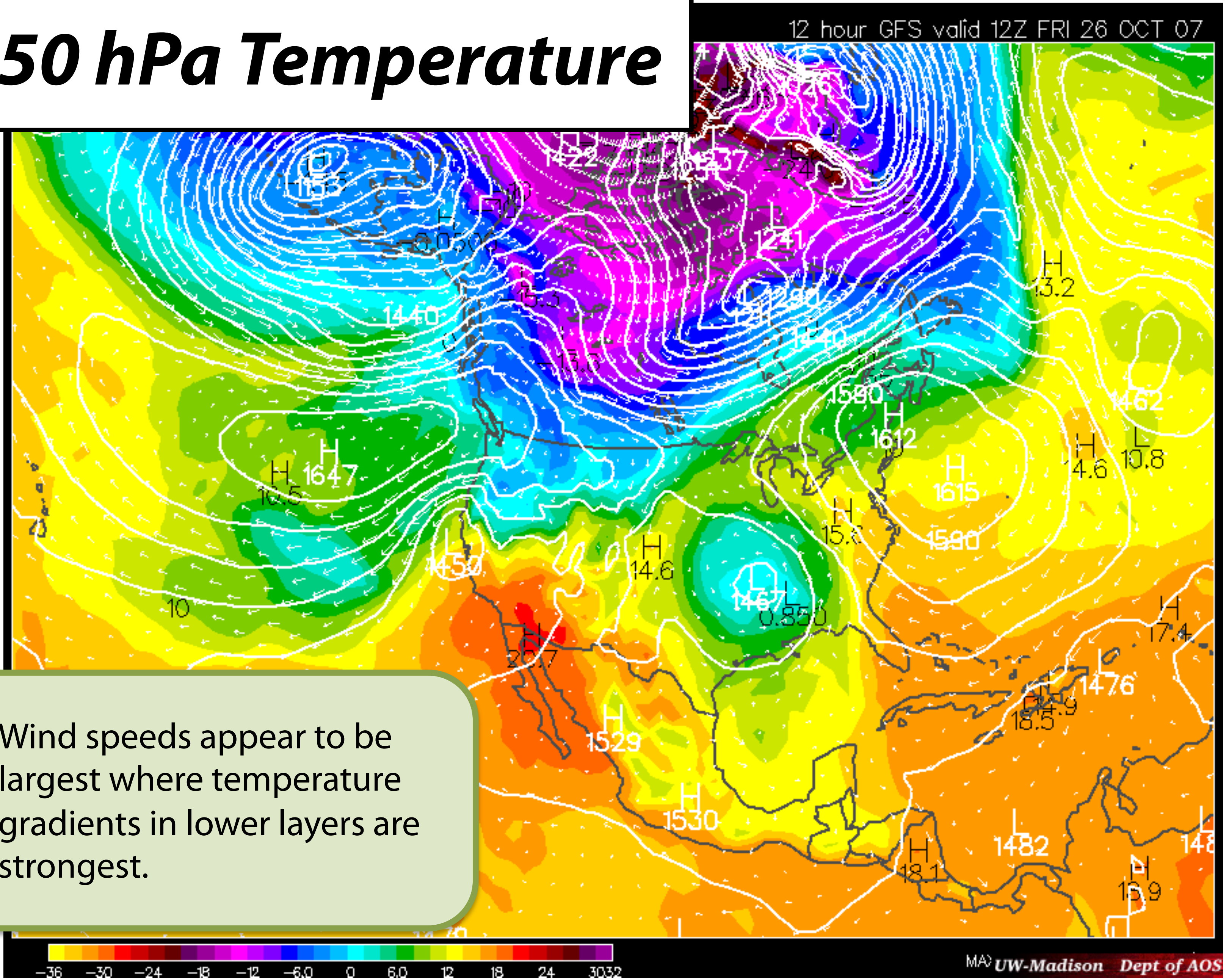
It turns out that there is a close link between **vertical wind shear** (vertical gradients of horizontal wind speed) and **layer thickness**, which is governed by temperature.

300 hPa Wind Speed



Question: Where are the strongest temperature gradients?

850 hPa Temperature



Wind speeds appear to be largest where temperature gradients in lower layers are strongest.

Thermal Wind

Definition: The **thermal wind** is a vector difference between the geostrophic wind at an upper level and a lower level.

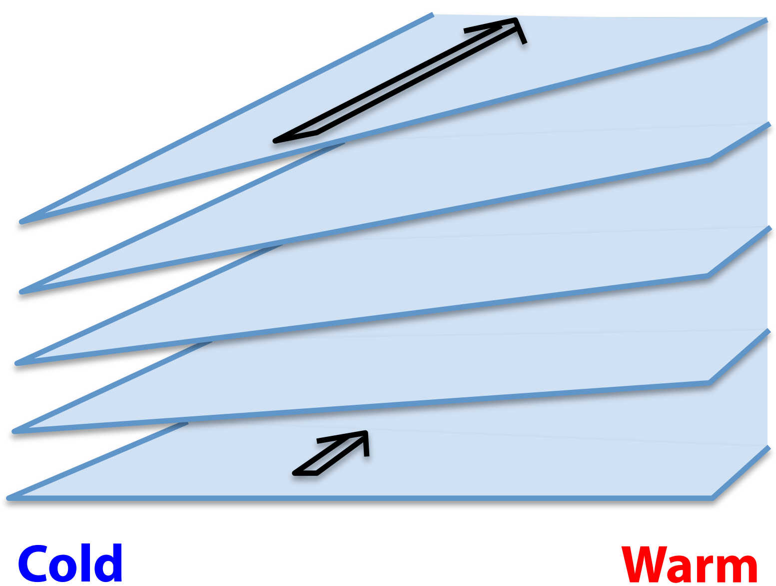
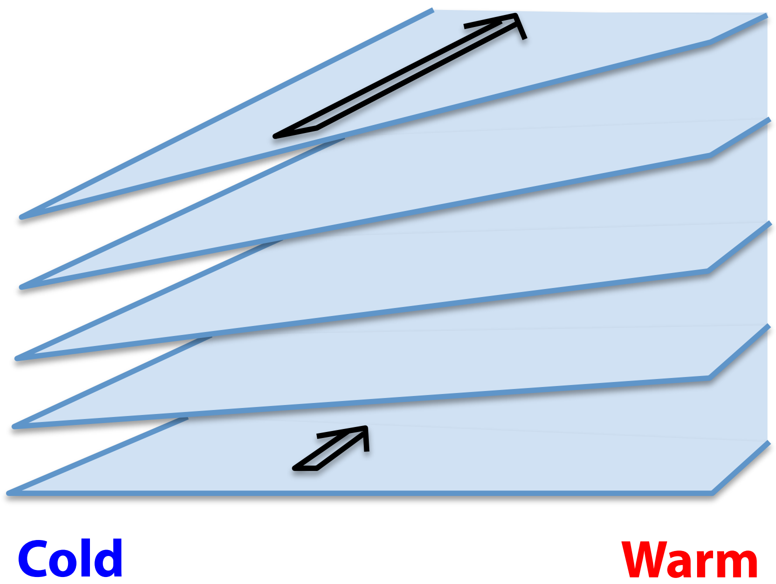


Figure: Thickness of layers related to temperature, causes a tilt in the pressure surfaces.

Change in magnitude of horizontal gradient of pressure then leads to vertical wind shear.

Thermal Wind

Emphasis: The thermal wind is not a real wind, but a vector difference.



The thermal wind vector points such that **cold air is to the left** and **warm air is to the right**, parallel to isotherms (in the northern hemisphere). Cold air is to the right and warm air is to the left in the southern hemisphere.

Geostrophic Wind

In Pressure Coordinates

Recall: The **geostrophic wind** is the component of the real wind which is governed by geostrophic balance. On constant height surfaces, it is defined to satisfy

$$u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y}$$
$$v_g = +\frac{1}{f\rho}\frac{\partial p}{\partial x}$$

In pressure coordinates, this relationship takes on the analogous form:

$$u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y}$$
$$v_g = +\frac{1}{f}\frac{\partial \Phi}{\partial x}$$

Geostrophic Wind

$$u_g = -\frac{1}{f}\left(\frac{\partial \Phi}{\partial y}\right)_p \qquad v_g = \frac{1}{f}\left(\frac{\partial \Phi}{\partial x}\right)_p$$

Differentiate with respect to p

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial y}\frac{\partial \Phi}{\partial p} \qquad \frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\frac{\partial \Phi}{\partial p}$$

Hydrostatic Relationship

$$\frac{\partial \Phi}{\partial p} = g\frac{\partial z}{\partial p} = -\frac{1}{\rho} = -\frac{RT}{p}$$

On constant p surfaces

Thermal wind relationship:

$$\frac{\partial u_g}{\partial p} = \frac{R}{pf}\left(\frac{\partial T}{\partial y}\right)_p \qquad \frac{\partial v_g}{\partial p} = -\frac{R}{pf}\left(\frac{\partial T}{\partial x}\right)_p$$

Thermal wind relationship:

$$\frac{\partial u_g}{\partial p} = \frac{R}{pf}\left(\frac{\partial T}{\partial y}\right)_p \qquad \frac{\partial v_g}{\partial p} = -\frac{R}{pf}\left(\frac{\partial T}{\partial x}\right)_p$$

This relationship links the horizontal temperature gradient with the vertical wind gradient (vertical shear).

Thermal Wind

Thermal wind relationship:

$$\frac{\partial u_g}{\partial p} = \frac{R}{pf}\left(\frac{\partial T}{\partial y}\right)_p$$

$$\frac{\partial v_g}{\partial p} = -\frac{R}{pf}\left(\frac{\partial T}{\partial x}\right)_p$$

The thermal wind itself is a vector difference.

Rewrite

$$\frac{\partial u_g}{\partial(\log p)} = \frac{R}{f}\left(\frac{\partial T}{\partial y}\right)_p \qquad \frac{\partial v_g}{\partial(\log p)} = -\frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p$$

Integrate

Thermal Wind

$$u_T = u_g(p_2) - u_g(p_1) = -\frac{R}{f}\left(\frac{\partial\langle T\rangle}{\partial y}\right)_p \log\left(\frac{p_1}{p_2}\right)$$

$$v_T = v_g(p_2) - v_g(p_1) = \frac{R}{f}\left(\frac{\partial\langle T\rangle}{\partial x}\right)_p \log\left(\frac{p_1}{p_2}\right)$$

Thermal Wind

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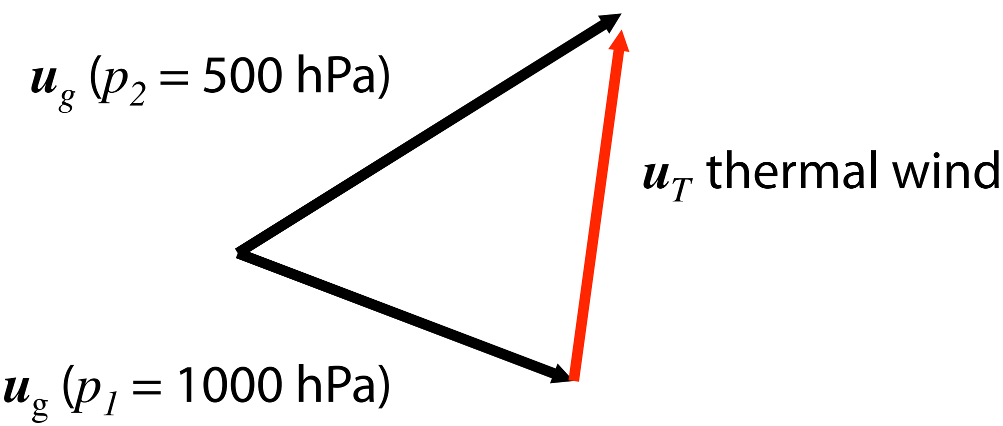
Example: Thermal wind v_T between 500 hPa and 1000 hPa



Thermal Wind

Alternate form of thermal wind, written in terms of geopotential height (obtained from hypsometric equation):

$$\mathbf{u}_T = \frac{R_d}{f}\mathbf{k} \times \nabla_p \langle T \rangle \log\left(\frac{p_1}{p_2}\right) = \frac{1}{f}\mathbf{k} \times \nabla_p (\Phi_2 - \Phi_1)$$

$$u_T = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1) \qquad v_T = \frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1)$$

Index "1" indicates lower level, "2" indicates upper level.

Thermal Wind

$$u_T \approx u_g(p_2) - u_g(p_1) = -\frac{R}{f}\left(\frac{\partial \langle T \rangle}{\partial y}\right)_p \log\left(\frac{p_1}{p_2}\right)$$

$$v_T \approx v_g(p_2) - v_g(p_1) = \frac{R}{f}\left(\frac{\partial \langle T \rangle}{\partial x}\right)_p \log\left(\frac{p_1}{p_2}\right)$$

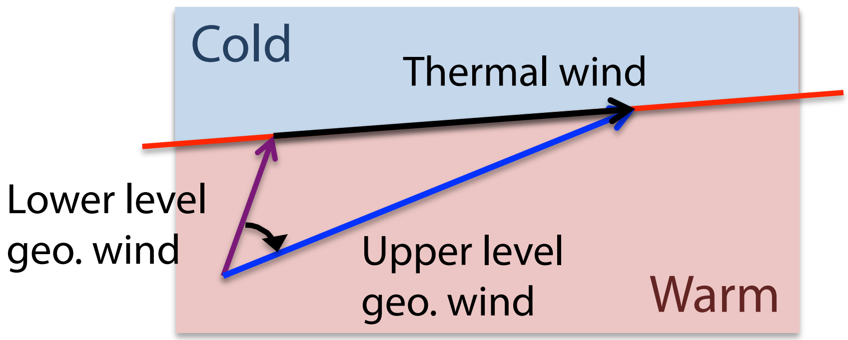
Note that thermal wind always points parallel to lines of constant layer temperature:

$$\mathbf{u}_T \cdot \nabla \langle T \rangle = \frac{R}{f} \log\left(\frac{p_1}{p_2}\right)\left[-\frac{\partial \langle T \rangle}{\partial y}\frac{\partial \langle T \rangle}{\partial x} + \frac{\partial \langle T \rangle}{\partial x}\frac{\partial \langle T \rangle}{\partial y}\right] = 0$$

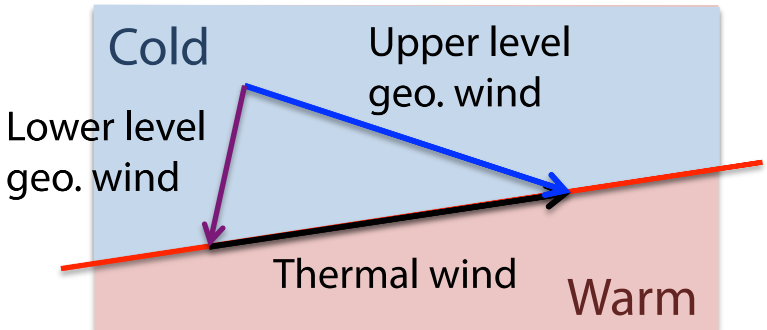
Thermal Wind

Thermal wind always points parallel to lines of constant temperature (and lines of constant layer thickness).

Definition: Veering winds are defined by clockwise rotation of the geostrophic wind with height and are associated with **warm air advection**.



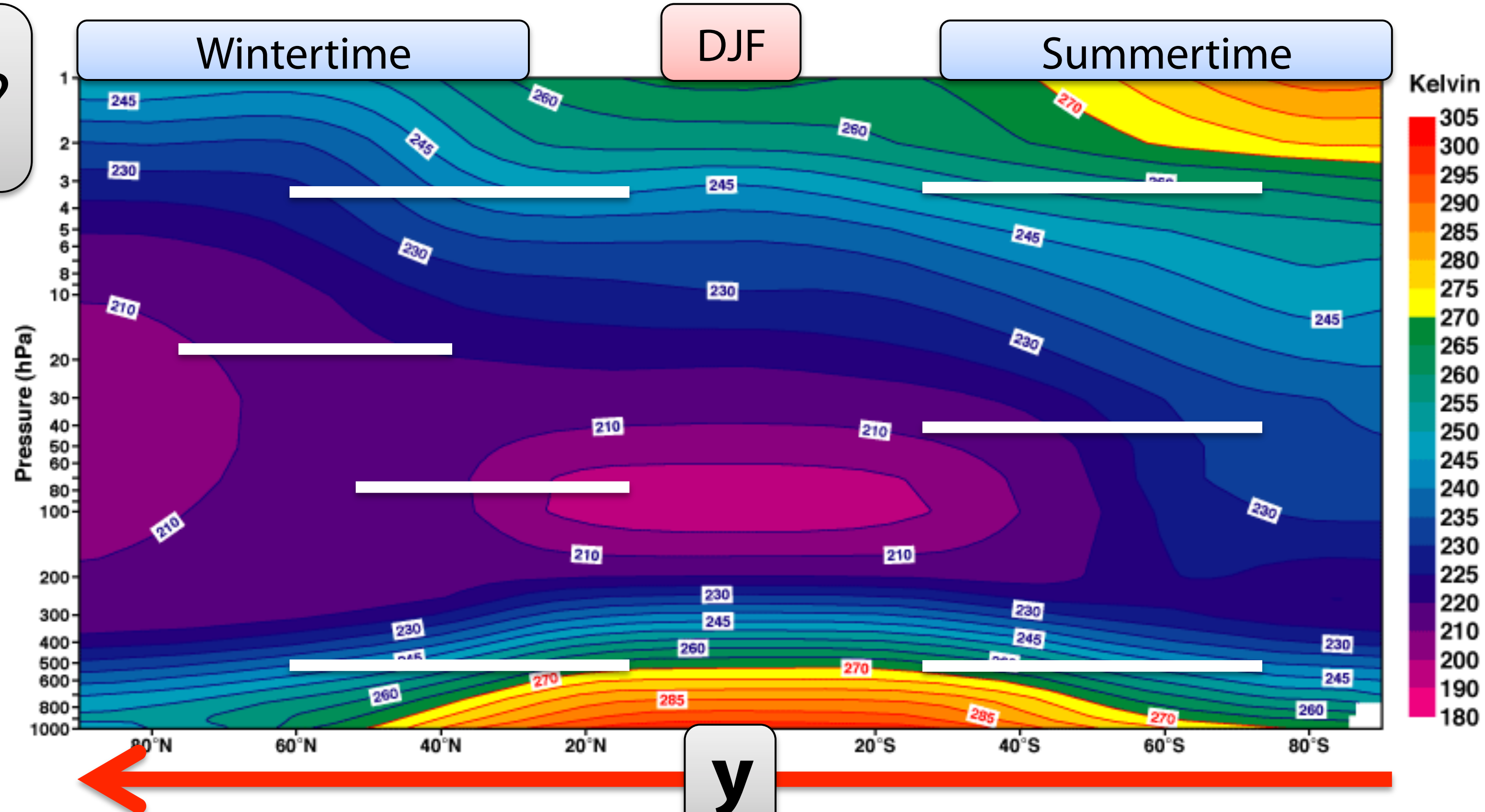
Definition: Backing winds are defined by counter-clockwise rotation of the geostrophic wind with height and are associated with **cold air advection**.



The thermal wind determines the relationship between meridional temperature gradients and zonal winds.

Question: Given zonal mean temperature below, where are zonal jets?

$$\frac{\partial T}{\partial y} ?$$



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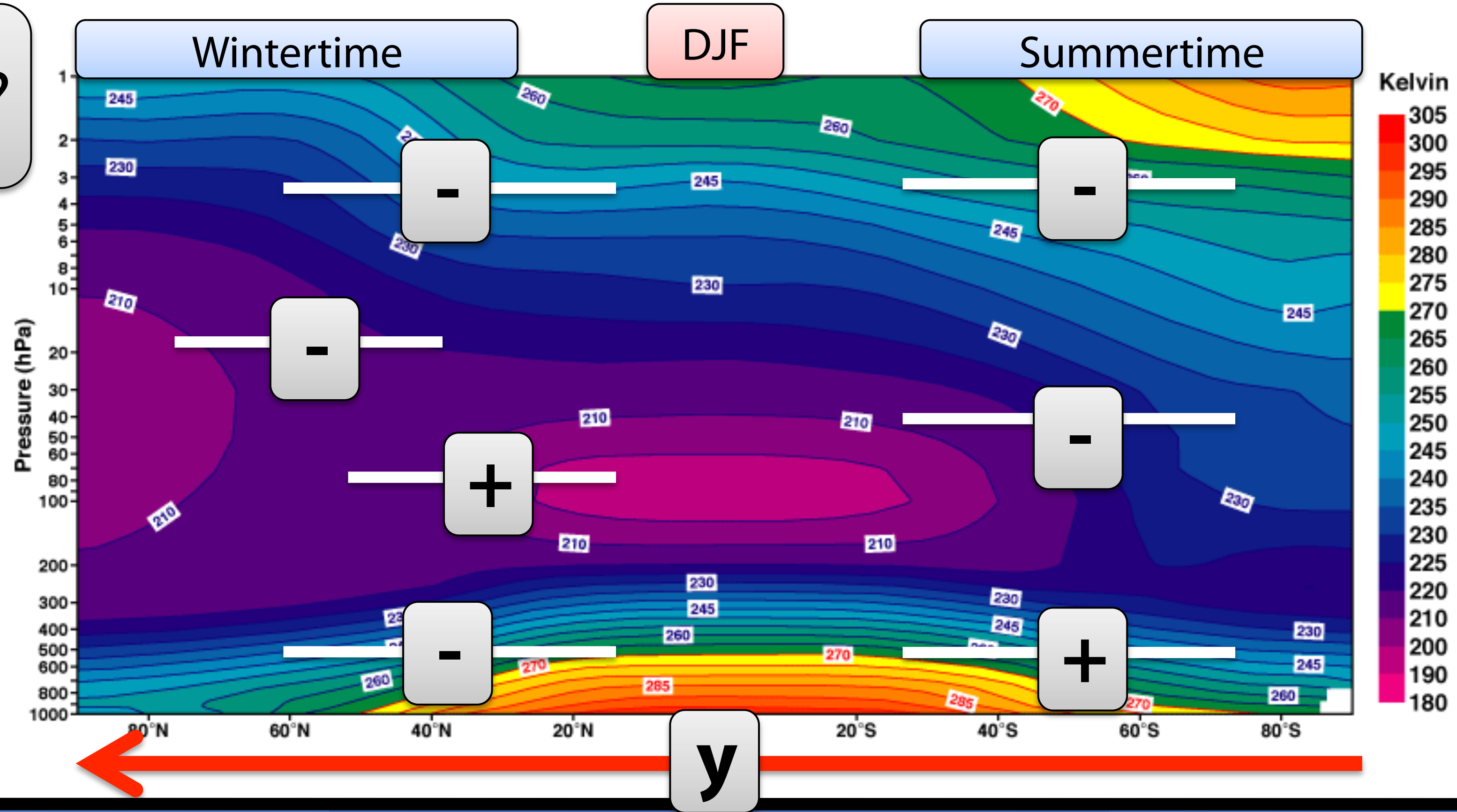
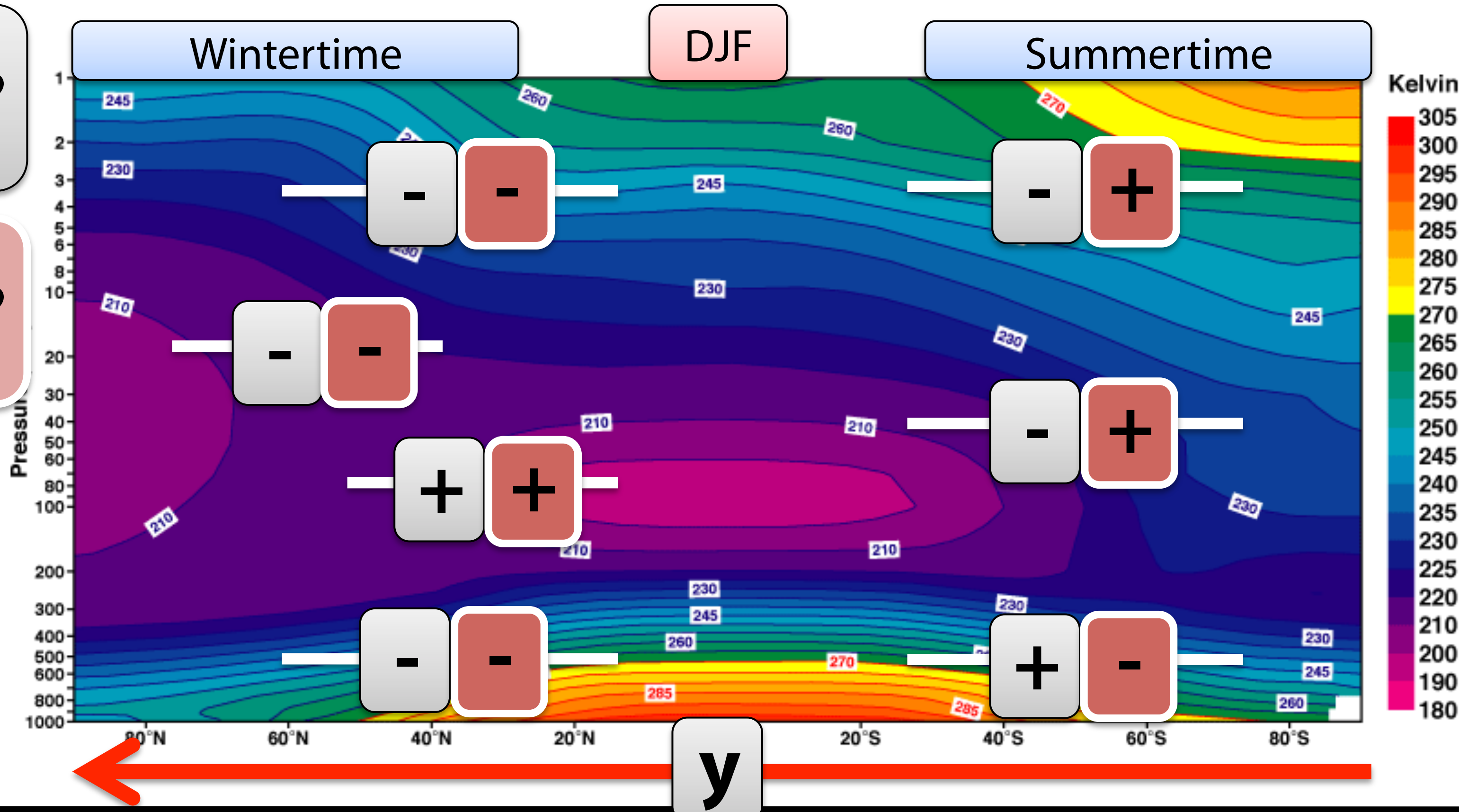


Question: Given zonal mean temperature below, where are zonal jets?

$$\frac{\partial u_g}{\partial p} = \frac{R}{pf}\left(\frac{\partial T}{\partial y}\right)_p$$

$\frac{\partial T}{\partial y}$?

$\frac{\partial u_g}{\partial p}$?

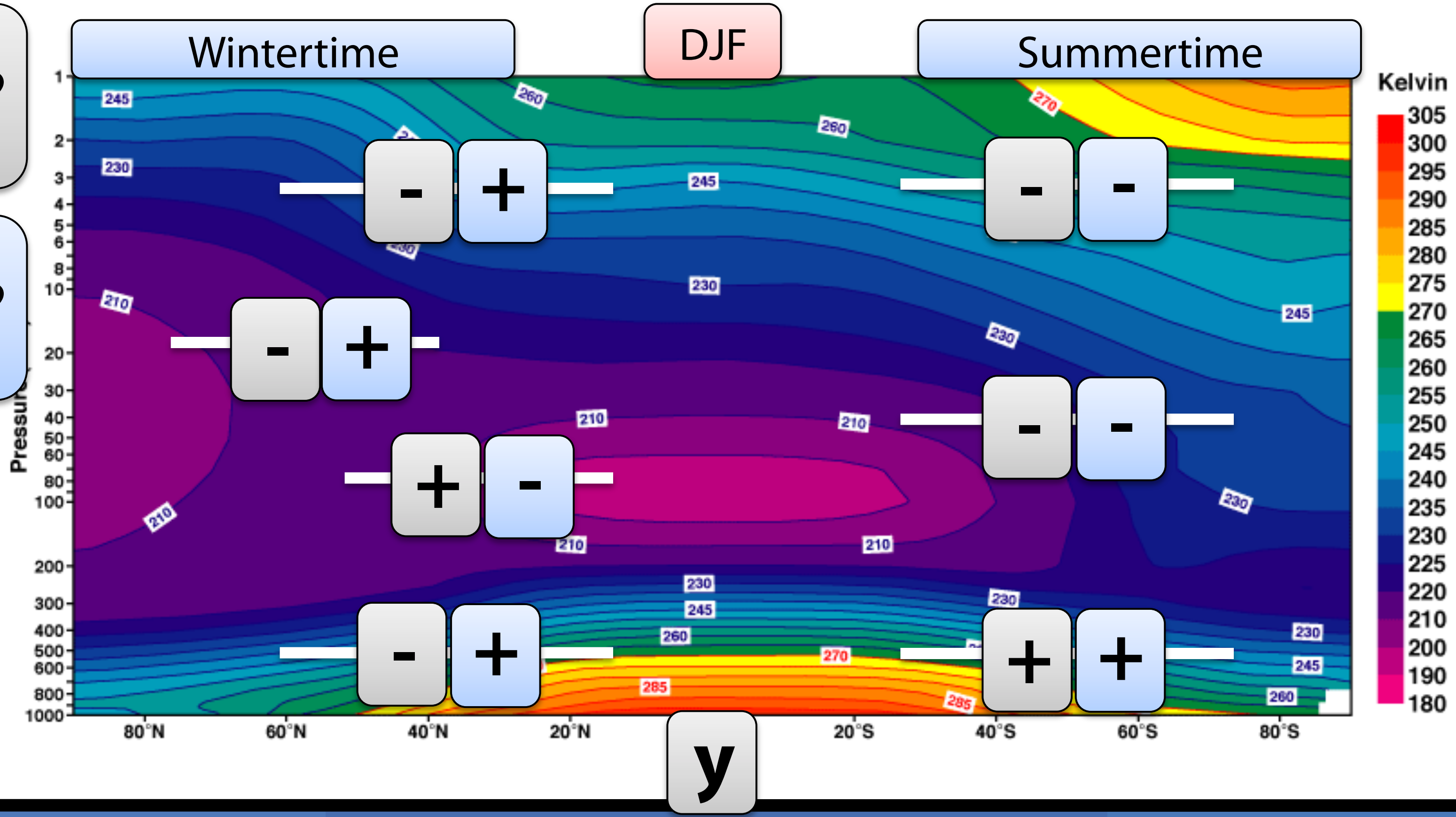


Question: Given zonal mean temperature below, where are zonal jets?

$$\frac{\partial u_g}{\partial p} = \frac{R}{pf}\left(\frac{\partial T}{\partial y}\right)_p \quad \frac{\partial u_g}{\partial z} \sim -\frac{\partial u_g}{\partial p}$$

$\dfrac{\partial T}{\partial y}$?

$\dfrac{\partial u_g}{\partial z}$?



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