Introduction to Atmospheric Dynamics
Chapter 1

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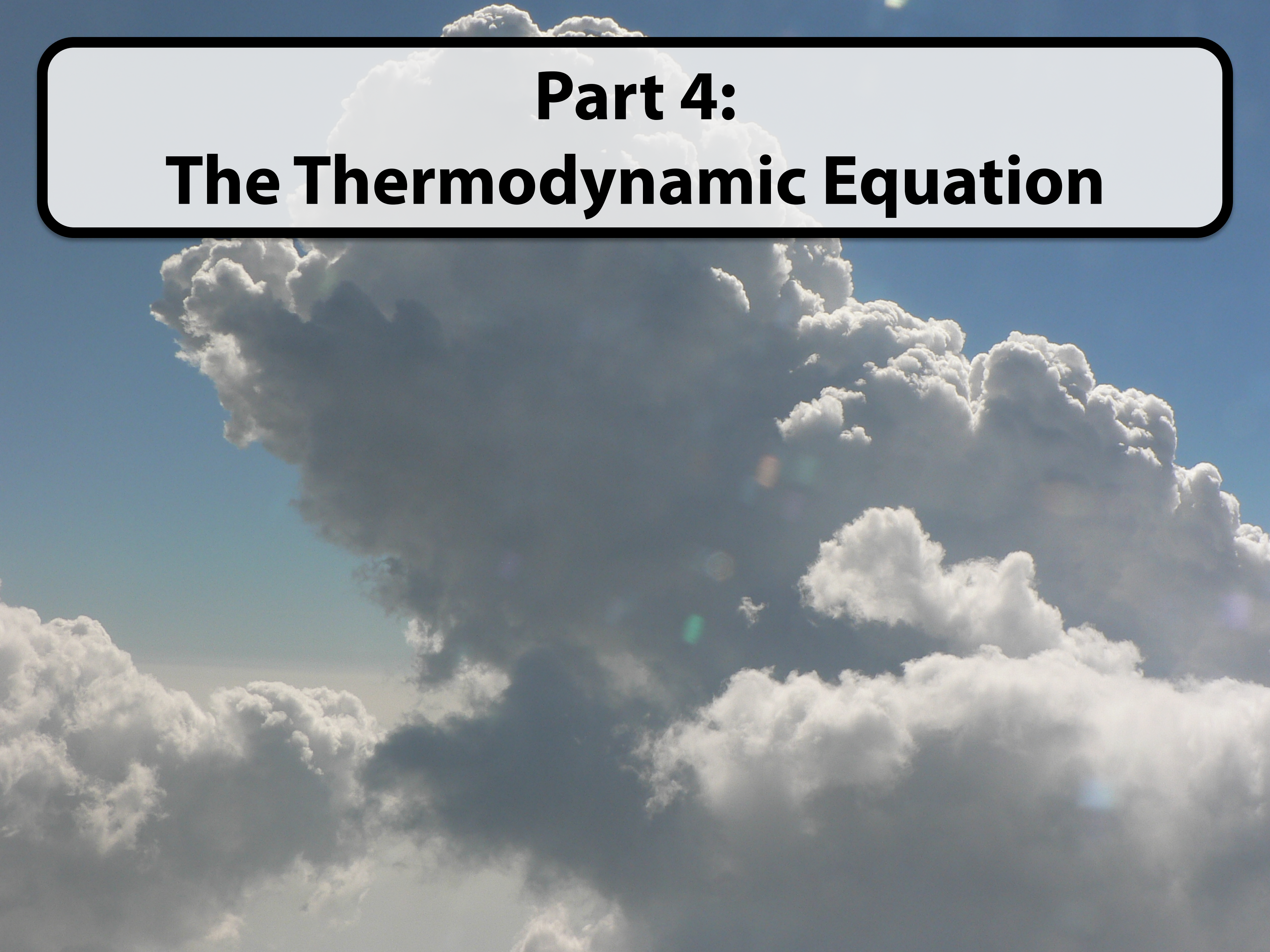
Part 4:
The Thermodynamic Equation



Question: What are the basic physical principles that govern the atmosphere?

Conservation of Energy: In a closed system, energy is not created nor destroyed. However, even in a closed system energy can change forms (thermal, kinetic, mechanical, potential).

This principle is enshrined in the **first law of thermodynamics**.

Thermodynamic Equation

First Law of Thermodynamics:
With **internal energy** U, **heating** δQ and **work** δW. States that the change in internal energy is equal to the heat added to the system plus the work done on the system

$$dU = \delta W + \delta Q$$

Internal energy is due to the vibrational kinetic energy of the molecules (temperature).

Thermodynamic Equation

Definition: The specific heat at constant volume (c_v) represents that amount of energy needed to raise the temperature of the air by one degree K if volume is held constant.

For dry air:

$$c_v = 717.5 \text{ J K}^{-1} \text{ kg}^{-1}$$

Definition: The specific heat at constant pressure (c_p) represents that amount of energy needed to raise the temperature of the air by one degree K if pressure is held constant.

For dry air:

$$c_p = 1004.5 \text{ J K}^{-1} \text{ kg}^{-1}$$

Ideal Gas Law

Atmosphere is composed of air, which is a mixture of gases (treated as an ideal gas). Below an altitude of 100 km, the atmosphere behaves as a fluid (under the ***continuum hypothesis***).

$$\boxed{p = \rho R_d T}$$

p	Pressure (Pa)
ρ	Density (kg/m^3)
R_d	Ideal gas constant for dry air
T	Temperature (Kelvin)

Ideal Gas Law: This is an equation which relates pressure, density and temperature and applies for many fluids where intermolecular attractions are negligible.

For dry air:

$$R_d = 287.0\ \mathrm{J\ K^{-1}\ kg^{-1}}$$

Thermodynamic Equation

Recall the first law of thermodynamics:

$$dU = \delta W + \delta Q$$

With internal energy U, heating Q and work W

$$dU = c_v dT \qquad \delta W = -p d\alpha \qquad \delta Q = J dt$$

Where $\alpha = \dfrac{1}{\rho}$ is the specific volume (volume / mass).

"divide" by dt :

$$\boxed{c_v \frac{dT}{dt} + p \frac{d\alpha}{dt} = J}$$

Heating / Cooling

Change in temperature of fluid parcel

Work done on fluid parcel

Thermodynamic Equation

$$\boxed{c_v \frac{dT}{dt} + p\frac{d\alpha}{dt} = J}$$

$$\boxed{d\alpha = d\left(\frac{1}{\rho}\right) = -\frac{1}{\rho^2}d\rho}$$

$$c_v \frac{dT}{dt} - \frac{p}{\rho^2}\frac{d\rho}{dt} = J$$

Ideal Gas Law

$$\boxed{p = \rho R_d T}$$

$$dp = R_d T d\rho + R_d \rho dT$$

$$(c_v + R_d)\frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt} = J$$

Thermodynamic Equation

$$(c_v + R_d)\frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt} = J$$

Alternative Form of **Thermodynamic Equation**

$$c_p = c_v + R_d$$

$$\boxed{c_p \frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt} = J}$$

Heating / Cooling

Change in temperature of fluid parcel

Work done on fluid parcel

Thermodynamic Equation for a Moving Air Parcel

$$c_v \frac{DT}{Dt} + p \frac{D\alpha}{Dt} = J$$

$$c_p \frac{DT}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} = J$$

J represents **diabatic sources** or **sinks** of energy:

- Radiation
- Latent heat release (condensation / evaporation)
- Thermal conductivity
- Frictional heating

For most large-scale motions, the amount of diabatic heating is relatively small.

Definition: Under **adiabatic flow** diabatic heating is exactly zero:

$$J = 0$$

Thermodynamics

$$c_p \frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = J$$

Ideal Gas Law

$$p = \rho R_d T$$

T_0, p_0 arbitrary

$$\frac{c_p}{T}\frac{DT}{Dt} - \frac{R}{p}\frac{Dp}{Dt} = \frac{J}{T}$$

$$c_p \frac{D}{Dt}\log(T/T_0) - R\frac{D}{Dt}\log(p/p_0) = \frac{J}{T}$$

$$\frac{D}{Dt}\log(T/T_0) - \frac{R}{c_p}\frac{D}{Dt}\log(p/p_0) = \frac{J}{c_p T}$$

$$\frac{D}{Dt}\log\left[\frac{T}{T_0}\left(\frac{p_0}{p}\right)^{R/c_p}\right] = \frac{J}{c_p T}$$

Thermodynamics

Three forms of the thermodynamic equation:

Form 1:

$$\boxed{c_v \frac{DT}{Dt} + p \frac{D\alpha}{Dt} = J}$$

Form 2:

$$\boxed{c_p \frac{DT}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} = J}$$

Form 3:

$$\boxed{\frac{D}{Dt} \log \left[\frac{T}{T_0} \left(\frac{p_0}{p} \right)^{R/c_p} \right] = \frac{J}{c_p T}}$$

Thermodynamics

$$\frac{D}{Dt}\log\left[\frac{T}{T_0}\left(\frac{p_0}{p}\right)^{R/c_p}\right] = \frac{J}{c_p T}$$

Adiabatic flow $J = 0$

$$\frac{D}{Dt}\left[T\left(\frac{p_0}{p}\right)^{R/c_p}\right] = 0$$

Definition: Potential temperature θ is the temperature an air parcel would have if its pressure were adiabatically adjusted to the reference pressure p_0.

$$\theta = T\left(\frac{p_0}{p}\right)^{R/c_p}$$

$$\boxed{\frac{D\theta}{Dt} = 0}$$

(potential temperature is conserved under adiabatic flow)

Thermodynamics

Potential Temperature

$$\theta = T \left(\frac{p_0}{p} \right)^{R/c_p}$$

In an adiabatic flow:

$$\frac{D\theta}{Dt} = 0$$

Typically p_0 represents sea-level pressure (10^5 Pa = 1000 hPa)

In this case, potential temperature is defined as the temperature an air parcel (with temperature T and pressure p) would have if it was adiabatically brought to sea-level pressure.

Potential temperature is closely associated with **entropy** (constant potential temperature is the same as constant entropy).

The Atmospheric Equations

$$\frac{Du}{Dt} - \frac{uv\tan\phi}{r} + \frac{uw}{r} = -\frac{1}{\rho r\cos\phi}\frac{\partial p}{\partial\lambda} + 2\Omega v\sin\phi - 2\Omega w\cos\phi + \nu\nabla^2 u$$

$$\frac{Dv}{Dt} + \frac{u^2\tan\phi}{r} + \frac{vw}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial\phi} - 2\Omega u\sin\phi + \nu\nabla^2 v$$

$$\frac{Dw}{Dt} - \frac{u^2+v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - g + 2\Omega u\cos\phi + \nu\nabla^2 w$$

$$\frac{D\rho}{Dt} = -\rho\cdot\nabla\mathbf{u}$$

$$c_p\frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = J$$

$$p = \rho R_d T$$

$$\frac{Dq_i}{Dt} = S_i$$

Material derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla$$